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Safety Assessment

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7 Safety Assessment

7.1 Definition of Safety and Safety Concepts

Structures have to be safe. However, there is no common understanding of the term "safety." Often the term "safety" is defined as a situation with a lower risk compared to an acceptable risk or as a situation "without any impending danger." Other definitions describe safety as "peace of mind." Whereas the first definition using the term "risk" is already based on a substitution, the later term using "peace of mind" is a better definition. The authors consider "safety" to be the result of an evaluation process of a certain situation. The evaluation can be carried out by every system that is able to perform a decision-making process, such as animals, humans, societies, or computers that use some algorithms. However, algorithms usually use some numerical representation, such as risk R, for the description of safety S:

$$\begin{aligned} &\text{existing } R \leq \text{permitted } R \rightarrow S \\ &\text{existing } R > \text{permitted } R \rightarrow \not{S} \end{aligned} \tag{7-1}$$

In contrast, the authors consider not only numerical presentations as results of decision-making processes, but also human feelings. Therefore, safety is understood here as a feeling. The decision-making process focuses mainly on preservation. Furthermore, the decision-making process deals with whether some resources have to be spent to decrease hazards and danger to an acceptable level. In other terms, "safety" is a feeling, which describes that no further resources have to be spent to decrease any threats. If one considers the term "no further resources have to be spent" as a degree of freedom of resources, one can define "safety" as a value of a function that includes the degree of freedom of resources. Furthermore, one can assume that the degree of freedom is related to some degree of distress and relaxation. Whereas in safe conditions relaxation occurs, in dangerous situations a high degree of distress is clearly reached.

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The possible shape of the function between degree of relaxation, which ranges from “danger” to “peace of mind,” and the value of the function as degree of freedom of resources is shown in Fig. 7-1. It is assumed here that the relationship is nonlinear, with at least one region of over proportional growth of the relative freedom of resources. In Figure 7-1, this region of over proportional growth is defined as the starting point of the safety region:

$$S = \{x \mid f''(x) = 0\} \tag{7-2}$$

However, the question still remains: where does the region of safety start since other points are possible? Such further points can be located either at regions of maximum curvature or at the point of inflection.

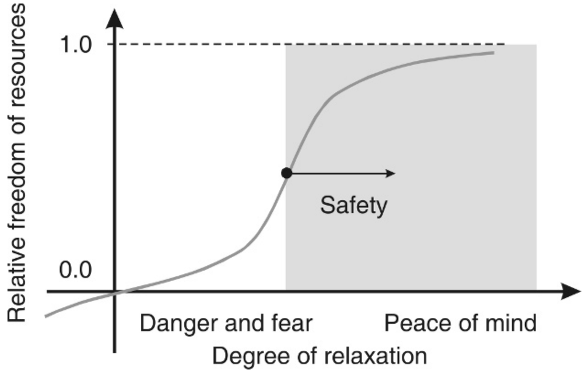


Fig. 7-1. Definition of “safety“ (Proske 2008a)

In general, safety is a general requirement for humans. This claim is manifested in many laws, like the human rights of the United Nations, the German constitution with the right to life and personal integrity, the Product Liability Act, civil code, or some administrative fiats. Codes of practice are administrative fiats and state the requirement that structures have to be safe (Proske 2008b). Also, in the sense of building laws, structures have to be safe and should not endanger public safety, life, and health. Especially in the codes, safety is understood as capability of structures to resist loads. Reliability is then understood as a measure to provide this capability in different engineering fields. Here, a change from the general qualitative statement to a quantitative statement becomes obvious. This is very important for the engineers: now the engineer is enabled to prove safety by computation. The reliability is meanly understood as probability of failure (Fig. 7-2). Risk, which would be an alternative measure of (un)safety, is only considered for accidental loads. Then, a comparison between different accidental loads and emergency situations is possible (Proske 2008b).

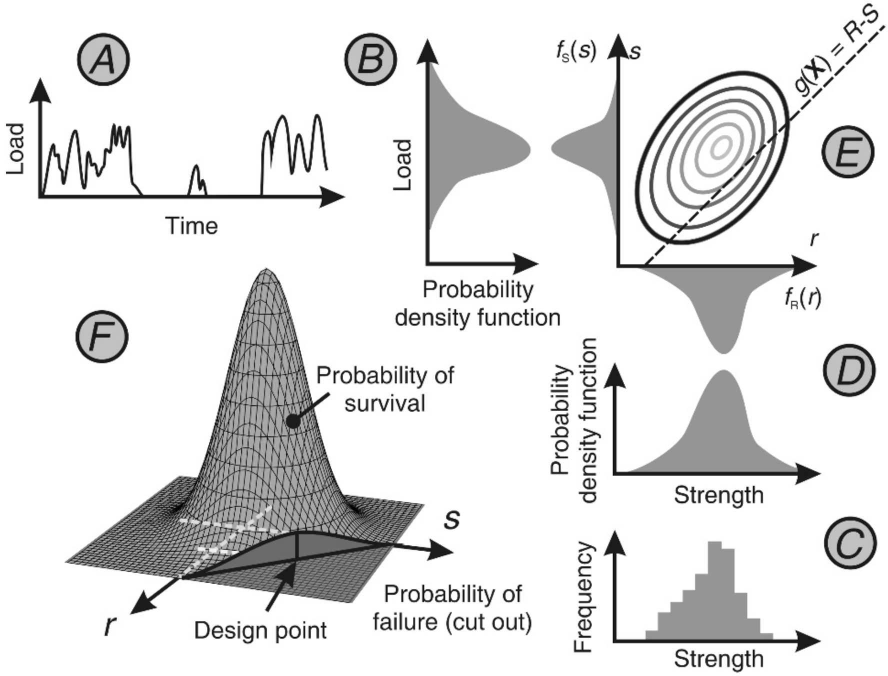


Fig. 7-2. Probability of failure for two random variables. First, (**A**) and (**C**) statistical data about the load and the strength are investigated. Then, a statistical investigation is carried out (**B**) and (**D**). Both distribution functions resulting from the statistical investigation are then merged to (**E**) further introducing a limit state function g(X). In (**F**), the two-dimensional distribution function is shown in three-dimensional illustrations of the probability of failure

The choice of probability of failure determines stochasticity as the basis for the exposure of indeterminacy and uncertainty. Other concepts, like fuzzy-sets, rough-sets, Grey numbers, or further mathematical techniques are not considered. However, research is carried out in this field. Figure 7-3 shows the different safety concepts for structures over time.

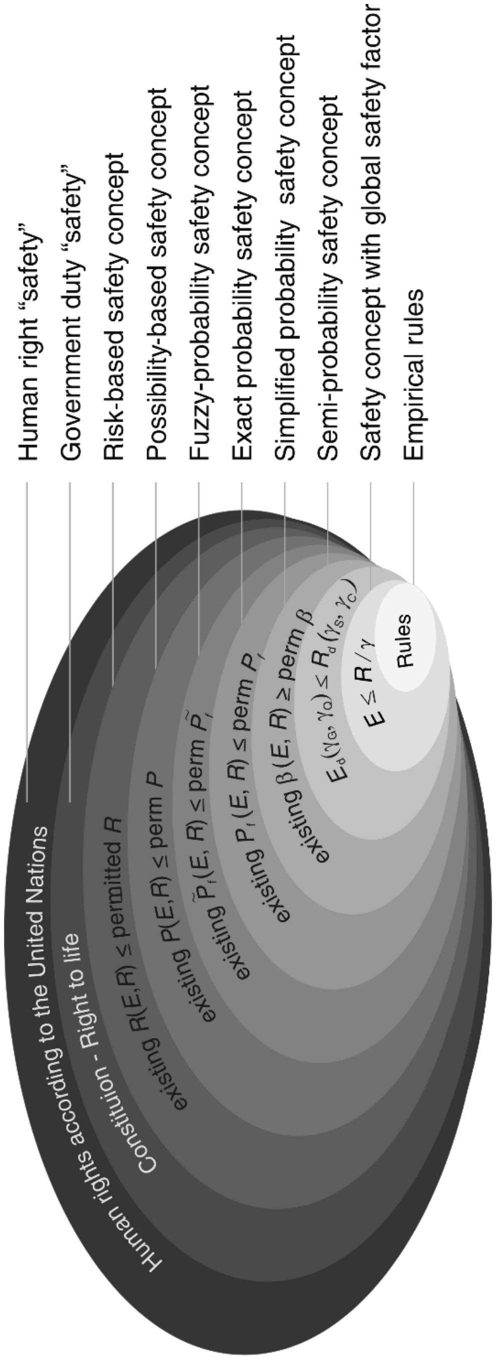


Fig. 7-3. Different safety concepts for structures

The current probabilistic or semiprobabilistic safety concept in structural engineering assumes that the exact value of many design variables is unknown. This uncertainty is based on

- Random aberrations of characteristic values of the structural resistance
- Random aberrations by transferring laboratory test results to the structure
- Random aberrations of cross-section sizes and other geometrical measures
- Geometric imperfections
- Random aberrations of internal forces like moments, shear forces, or axial forces
- Inherent uncertainties in the choice of characteristic value of loads
- Differences in the models for the loads

However, the stochastic models do not consider systematical errors like computational errors in structural design processes or bad workmanship. Such errors have to be avoided by control mechanisms (DIN 1055-100 1999).

7.2 Probabilistic Safety Concept

7.2.1 Introduction

First proposals about probabilistic-based safety concepts can be found by Mayer (1926) in Germany and Chocialov (1929) in the Soviet Union (Murzewski 1974). In the third decade of the 20th century, the number of people working in that field had already increased, just to mention Streleckij (1935) in the Soviet Union, Wierzbicki (1936) in Poland, and Prot (1936) in France (Murzewski 1974). Already in 1944 in the Soviet Union, the introduction of the probabilistic safety concept for structures had been forced by politicians (Tichý 1976). The development of probabilistic safety concepts in general experienced a strong impulse during and after World War II, not only in the field of structures but also in the field of aeronautics. In 1947, Freudenthal (1947) published his famous work about the safety of structures. Until now, a model code for the probabilistic safety concept of structures has been published by the JCSS (2004).

The probability of failure p_f as proof measure for safety is computed as function of the design values x. It can be referred to one year or the lifetime of the structures:

$$p_f = \int \dots \int_{g(\mathrm{X}) \le 0} f_{\mathrm{X}}(x)dx \tag{7-3}$$

$$p_f(n) = 1 - (1 - p_f)^n . \tag{7-4}$$

The safety index is defined as the inverse Gauss standard distribution of the probability of failure:

$$\beta = -\Phi^{-1}(p_f). \tag{7-5}$$

Results are given in Table 7-1. The integration of the probability of failure volume can then be transferred into an optimization task to determine the safety index. This is shown in Figs. 7-4 and 7-5.

The explained safety concept can be found in many regulations, such as Eurocode 1 (1994), DIN 1055-100 (1999), GruSiBau (1981), and JCSS Modelcode (2004). In these regulations, goal values for safety indexes can also be found. These values are then the basis for the estimation of safety factors, which are introduced for practical reasons.

Table 7-1. Conversion of probability of failure to safety index

Probability of failure	10^{-12}	10^{-11}	10^{-10}	10^{-9}	10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	0.5
Safety index	7.03	6.71	6.36	5.99	5.61	5.19	4.75	4.26	3.72	3.09	2.33	1.28	0.0

7.2.2 First-order Reliability Method (FORM)

Since structures should feature a low probability of failure, the computation of the multidimensional probability may be simplified due to the low value. The simplification explained in this section increases the speed of the computation tremendously compared to a numerical integration of a multidimensional space.

In general, the simplification is based on the transfer of the integration of a multidimensional volume into an extreme value task. The result of this extreme value computation is the substitute measure safety index. The safety index itself describes the shortest distance between the origin in a standard normal distributed space and the limit state function, $g(\mathbf{X})$. A relationship between the probability of failure volume and the distance expressed by the safety index exists in this space (Table 7-1).

The standard normal distribution is characterized by a normal distribution with a mean value of 0 and a standard deviation of 1. The general assumption of this procedure is the transformability of all arbitrary random distribution functions into standard normal distribution functions. The second assumption is the linearization of the limit state function. The linearization gave the following name for the technique: first-order reliability method (FORM). The point of linearization on the limit state function is the so-called design point. This point is characterized by maximum probability of failure at the limit state function.

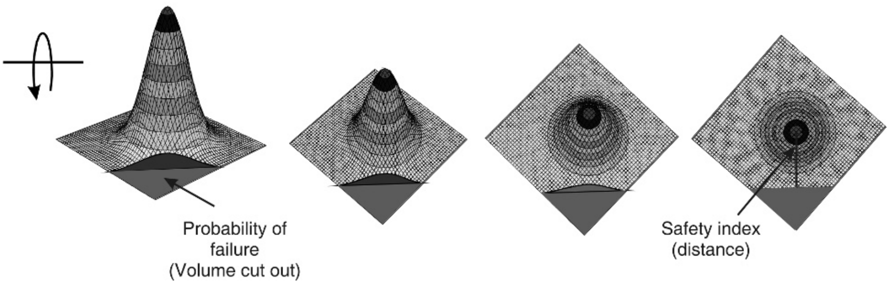


Fig. 7-4. Transfer of the probability volume into an extreme value computation

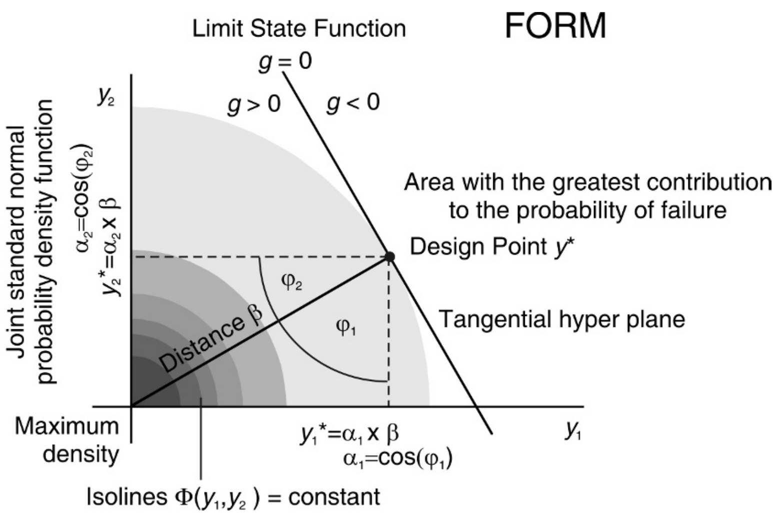


Fig. 7-5. Visualization of FORM

However, a FORM computation not only delivers the safety index as result, but also measures which may be useful to compute partial safety factors, characteristic values, and design values. These values can be found in many codes of practice and indicate the strong relationship between the codes and this probabilistic safety concept. Therefore, current safety concepts are called semiprobabilistic safety concepts.

In the following paragraphs, the FORM-methodology will be introduced in detail. The method is often called the Rackwitz-Fießler (Fießler et al. 1976) algorithm or normal tail approximation. In general, the concept is based on the fundamental work by Hasofer and Lind (1974).

In the procedure, first the non-normal distributed random variables have to be transferred into normal random variables. The following formulas will be used

$$f_{x_i}(x_i^*) = \frac{1}{\sigma_{x_i}^*}\varphi\left(\frac{x_i^* - m_{x_i}^*}{\sigma_{x_i}^*}\right) \tag{7-6}$$

$$F_{x_i}(x_i^*) = \Phi\left(\frac{x_i^* - m_{x_i}^*}{\sigma_{x_i}^*}\right) \tag{7-7}$$

with x_i^* as design point, $m_{x_i}^*$ as mean value, and $\sigma_{x_i}^*$ as standard deviation of the normal distribution. Since the normal distribution should be used as an approximation of the original distribution, mean value and standard deviation have to be computed by rearranging the formulas

$$\sigma_{x_i}^* = \frac{1}{f_{x_i}(x_i^*)}\varphi(\Phi^{-1}(F_{x_i}(x_i^*))) \tag{7-8}$$

$$m_{x_i}^* = x_i^* - \sigma_{x_i}^*\Phi^{-1}(F_{x_i}(x_i^*)) . \tag{7-9}$$

After that, an iteration cycle with the following steps is started:

1. Define an iteration counter $k = 0$ and chose a design point for the first iteration.
2. Transfer all non-normal distributed random variables into normal distributed random variables according to the following equations, with $i = 1, 2, \ldots, m$ and m as number of random variables considered:

$$\sigma_{x_i}^{*(k)} = \frac{1}{f_{x_i}(x_i^{(k)})}\varphi(\Phi^{-1}(F_{x_i}(x_i^{(k)}))) \tag{7-10}$$

$$m_{x_i}^{*(k)} = x_i^{(k)} - \sigma_{x_i}^{(k)}\Phi^{-1}(F_{x_i}(x_i^{(k)})) . \tag{7-11}$$

3. Compute the value of $x_i^{(k)}$ in the standardized space $y_i^{(k)}$:

$$y_i^{(k)} = \frac{x_i^{(k)} - m_{x_i}^{*(k)}}{\sigma_{x_i}^{*(k)}} \,. \tag{7-12}$$

4. Compute the limit state function and the first derivative at $y_i^{(k)}$:

$$h(\mathbf{y}^{(k)}) = g(\mathbf{x}^{(k)}) \tag{7-13}$$

$$\left.\frac{\partial h}{\partial y_i}\right|_{\mathbf{y}=\mathbf{y}^{(k)}} = \left.\frac{\partial g}{\partial x_i}\right|_{\mathbf{x}=\mathbf{x}^{(k)}} \cdot \frac{\partial x_i}{\partial y_i} = \left.\frac{\partial g}{\partial x_i}\right|_{\mathbf{x}=\mathbf{x}^{(k)}} \cdot \sigma_i^{*(k)} \,. \tag{7-14}$$

5. Compute the coefficients of the tangential hyperplane at $h(y) = 0$ at point $y_i^{(k)}$:

$$\alpha_i^{(k)} = \frac{\left.\frac{\partial h}{\partial y_i}\right|_{\mathbf{y}=\mathbf{y}^{(k)}}}{\left(\sum_{j=1}^{m}\left(\left.\frac{\partial h}{\partial y_j}\right|_{\mathbf{y}=\mathbf{y}^{(k)}}\right)^2\right)^{1/2}} \tag{7-15}$$

$$\delta^{(k)} = \frac{h(y^{(k)}) - \sum_{j=1}^{m} y_j^{(k)} \left.\frac{\partial h}{\partial y_j}\right|_{\mathbf{y}=\mathbf{y}^{(k)}}}{\left(\sum_{j=1}^{m}\left(\left.\frac{\partial h}{\partial y_j}\right|_{\mathbf{y}=\mathbf{y}^{(k)}}\right)^2\right)^{1/2}} \,. \tag{7-16}$$

6. Compute a new estimation of the design point in the original space for $i = 1, 2, \ldots, m$:

$$x_i^{(k+1)} = m_{x_i}^{*(k)} - \alpha_i^{*(k)} \cdot \sigma_{x_i}^{*(k)} \cdot \delta^{(k)} \,. \tag{7-17}$$

7. Verify if $x_i^{(k+1)} \approx x_i^{(k)}$. If it is fulfilled, then the design point has been found and the safety index is $\beta = \delta$ with $h(0) > 0$. If it is not fulfilled, then the iteration starts again with Step 2.

This method is very practicable and has experienced a major spreading. It gives fast and accurate results if the probability of failure is small, the random distribution functions do not diverge too strong from the normal distribution, and the limit state function does not show a strong curvature.

7.2.3 Second-order Reliability Method (SORM)

7.2.3.1 Breitung's method

If the limit state function shows a strong curvature, then the curvature has to be considered in the computation of the safety index (Fig. 7-6). This can be done by the second-order reliability method. Here, the curvature of the limit state function is approximated using

$$h(\mathbf{y}) = h(\mathbf{y}^*) + (\mathbf{y}-\mathbf{y}^*)^T \cdot \nabla h(\mathbf{y}^*) + \frac{1}{2}(\mathbf{y}-\mathbf{y}^*)^T \cdot \mathbf{B}_y \cdot (\mathbf{y}-\mathbf{y}^*) = 0\,. \tag{7-18}$$

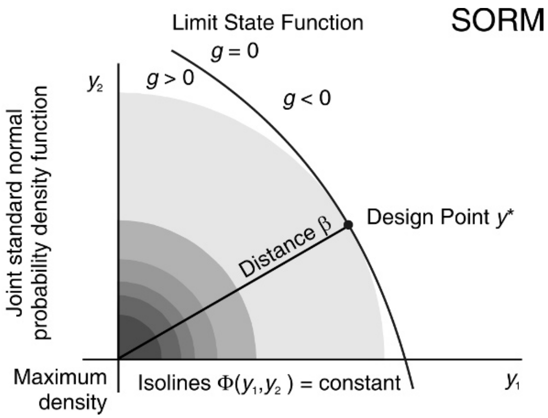


Fig. 7-6. Visualization of SORM

$\mathbf{B}_y$ is the matrix of the second and mixed derivates from $h(\mathbf{y})$ in the standardized space at the design point. Breitung (1984) has introduced the following equation with a_i and $i = 1, 2, \dots, m$-1 as principal curvature of h at the design point in the standard normal space

$$P_f = \Phi(-\beta)\prod_{i=1}^{m-1}(1-\beta \cdot a_i)^{-1/2}\ . \tag{7-19}$$

However, the computation of the principal curvatures represents the major part of this method. To compute the principal curvatures, a rotation of the coordinate system is required. The rotation requires orthogonalizing using the Schmidt process.

The old and new coordinates are linked as follows:

$$\mathbf{y} = \mathbf{D} \cdot \mathbf{u}\ . \tag{7-20}$$

After the rotation the new coordinates can be computed as

$$\mathbf{u} = \mathbf{D}^T \cdot \mathbf{y} . \tag{7-21}$$

The matrix **D** consists of

$$\mathbf{D} = (\mathbf{d}_1, \mathbf{d}_2, ..., \mathbf{d}_m)^T \tag{7-22}$$

with

$$\begin{aligned} \mathbf{d}_1 &= \alpha \\ \mathbf{d}_k &= \frac{f_k}{|f_k|} \end{aligned} \tag{7-23}$$

and

$$f_k = \mathbf{e}_k - \sum_{l=1}^{k-1} (\mathbf{e}_k^T \mathbf{d}_l) \mathbf{d}_l \tag{7-24}$$

$$k = 2,\ 3,\ ..., m .$$

and $\mathbf{e}_k$ is the *k*th unit vector. In the new system,

$$\mathbf{u}^* = \mathbf{D}^T \mathbf{y}^* = (\beta, 0, 0, ..., 0)^T \tag{7-25}$$

and

$$\nabla g_u = \mathbf{D} \cdot \nabla h = \left(\frac{\partial g_u}{\partial u_1}, 0, 0, ..., 0 \right)^T . \tag{7-26}$$

$$g_u(\mathbf{u}^*) = 0 \tag{7-27}$$

$$\mathbf{B}_u = \mathbf{D}^T \mathbf{B}_y \mathbf{D} \tag{7-28}$$

Taylor's theorem becomes

$$(u_1 - u_1^*) \cdot \frac{\partial g_u}{\partial u_1} + \frac{1}{2} (\mathbf{u} - \mathbf{u}^*)^T \cdot \mathbf{B}_u \cdot (\mathbf{u} - \mathbf{u}^*) = 0 \quad . \tag{7-29}$$

The principal curvatures are the roots of the following equation:

$$\det \left(\frac{\hat{\mathbf{B}}_u}{\partial g_u / \partial u_1} - a \cdot \mathbf{I} \right) = 0 \quad , \tag{7-30}$$

When $\hat{\mathbf{B}}_u$ is the matrix of the second and mixed derivates, and can be derived by deleting the first line and first column from $\mathbf{B}_u$, I is the identity matrix.

The approximation by Breitung gives good results, if the curvature of the limit state function still is not too strong and the safety index is small.

7.2.3.2 Tvedt's method

Tvedt (1988) has introduced an extension of Breitung's formulae:

$$\gamma = 1 - P_f \tag{7-31}$$

$$\gamma = 1 - (A_1 + A_2 + A_3) \tag{7-32}$$

with

$$A_1 = \Phi(-\beta)\prod_{j=1}^{n-1}(1+\beta \cdot a_j)^{-1/2} \tag{7-33}$$

$$A_2 = [\beta \cdot \Phi(-\beta) - \varphi(\beta)] \cdot \left\{ \prod_{j=1}^{n-1}(1+\beta \cdot a_j)^{-1/2} - \prod_{j=1}^{n-1}(1+(\beta+1)a_j)^{-1/2} \right\} \tag{7-34}$$

$$\begin{aligned} A_3 &= (\beta+1)[\beta \cdot \Phi(-\beta) - \varphi(\beta)] \\ &\cdot \left\{ \prod_{j=1}^{n-1}(1+\beta \cdot a_j)^{-1/2} - \mathrm{Re}\left[\prod_{j=1}^{n-1}(1+(\beta+\mathrm{i}) \cdot a_j)^{-1/2} \right] \right\}. \end{aligned} \tag{7-35}$$

The extension of Breitung's formula provides accurate results with low and semi-low probabilities of failure. High probabilities of failure and negative curvatures are not covered by the method. A further improvement was done by the so-called Tvedt's exact solution for parabolic limit state function. Here, the size of the probability of failure is not limited:

$$\gamma = 1 - P_f \tag{7-36}$$

$$\gamma = 0.5 + \frac{1}{\pi}\int_0^{\infty} \sin\left[\beta \cdot \theta + \frac{1}{2}\sum_{j=1}^{n-1}\arctan(a_j\theta) \right] \frac{\exp(-1/2\theta^2)}{\theta \prod_{j=1}^{n-1}(1+a_j^2\theta^2)^{1/4}} \mathrm{d}\theta . \tag{7-37}$$

The disadvantage of this method is the requirement of a numerical integration.

7.2.3.3 Method by Köylüoglu and Nielsen

Additionally, Köylüoglu and Nielsen (1994) have tried to improve the accuracy of SORM methods for higher probabilities of failure. They also formulate

$$\gamma = 1 - P_f \,. \tag{7-38}$$

Assuming that all curvatures a_i are positive, then

$$\begin{aligned}
\gamma = 1 - \Phi(-\beta)\prod_{j=1}^{n-1}\frac{1}{\sqrt{1+a_j/c_{0,1}}}\\
\cdot\Bigg\{1 + \frac{1}{2}c_{1,1}\sum_{k=1}^{n-1}\frac{a_k}{1+a_k/c_{0,1}} + \frac{1}{4}c_{2,1}\left[\left(\sum_{k=1}^{n-1}\frac{a_k}{1+a_k/c_{0,1}}\right)^2 + 2\sum_{k=1}^{n-1}\left(\frac{a_k}{1+a_k/c_{0,1}}\right)^2\right]\\
+\frac{1}{8}c_{3,1}\left[\left(\sum_{k=1}^{n-1}\frac{a_k}{1+a_k/c_{0,1}}\right)^3 + 2\left(\sum_{k=1}^{n-1}\frac{a_k}{1+a_k/c_{0,1}}\right)\left(\sum_{k=1}^{n-1}\left(\frac{a_k}{1+a_k/c_{0,1}}\right)^2\right)\right.\\
\left.+12\sum_{k=1}^{n-1}\left(\frac{a_k}{1+a_k/c_{0,1}}\right)^3\right] + \ldots\Bigg\}.
\end{aligned} \tag{7-39}$$

If all curvatures a_i are negative, then

$$\begin{aligned}
\gamma = 1 - \Phi(+\beta)\prod_{j=1}^{n-1}\frac{1}{\sqrt{1-a_j/c_{0,2}}}\\
\cdot\Bigg\{1 + \frac{1}{2}c_{1,2}\sum_{k=1}^{n-1}\frac{a_k}{1-a_k/c_{0,2}} + \frac{1}{4}c_{2,2}\left[\left(\sum_{k=1}^{n-1}\frac{a_k}{1-a_k/c_{0,2}}\right)^2 + 2\sum_{k=1}^{n-1}\left(\frac{a_k}{1+a_k/c_{0,2}}\right)^2\right]\\
+\frac{1}{8}c_{3,2}\left[\left(\sum_{k=1}^{n-1}\frac{a_k}{1+a_k/c_{0,2}}\right)^3 + 2\left(\sum_{k=1}^{n-1}\frac{a_k}{1+a_k/c_{0,2}}\right)\left(\sum_{k=1}^{n-1}\left(\frac{a_k}{1+a_k/c_{0,2}}\right)^2\right)\right.\\
\left.+12\sum_{k=1}^{n-1}\left(\frac{a_k}{1+a_k/c_{0,2}}\right)^3\right] + \ldots\Bigg\}.
\end{aligned} \tag{7-40}$$

The generalized form is given as follows:

$$\gamma = \Phi(\beta) + \Phi(-\beta)\left[\prod_{j=m}^{n-1}\frac{1}{\sqrt{1-a_j/d_{0,2}}}\left\{1+\frac{1}{2}d_{1,2}\sum_{k=m}^{n-1}\frac{a_k}{1-a_k/c_{0,2}}+\ldots\right\}\right.$$
$$\cdot\left\{1-\prod_{j=1}^{m-1}\frac{1}{\sqrt{1+a_j/c_{0,1}}}\left\{1+\frac{1}{2}c_{1,1}\sum_{k=m}^{m-1}\frac{a_k}{1+a_k/c_{0,1}}+\ldots\right\}\right\}\Bigg]$$
$$-\Phi(\beta)\left[\prod_{j=1}^{m-1}\frac{1}{\sqrt{1+a_j/d_{0,1}}}\left\{1+\frac{1}{2}d_{1,1}\sum_{k=1}^{m-1}\frac{a_k}{1+a_k/c_{0,1}}+\ldots\right\}\right.$$
$$\left.\cdot\left\{1-\prod_{j=m}^{n-1}\frac{1}{\sqrt{1-a_j/c_{0,2}}}\left\{1+\frac{1}{2}c_{1,2}\sum_{k=1}^{n-1}\frac{a_k}{1-a_k/c_{0,2}}+\ldots\right\}\right\}\right]. \tag{7-41}$$

Based on the cut-off of the terms, different approximation formulations can be derived. For one term with positive curvature, the coefficient becomes

$$c_{0,1} = \frac{\Phi(-\beta)}{\varphi(\beta)} \tag{7-42}$$

and $c_{1,1} = c_{2,1} = \ldots = 0$. If two terms are chosen, then the coefficients can be computed as

$$c_{0,1} = \frac{\Phi(-\beta)}{\varphi(\beta)}\left(\frac{1}{1+\sqrt{1-\beta\Phi(-\beta)/\varphi(\beta)}}\right) \tag{7-43}$$

$$c_{1,1} = \frac{\varphi(\beta)}{\Phi(-\beta)}\sqrt{1-\frac{\beta\Phi(-\beta)}{\varphi(\beta)}} \tag{7-44}$$

and $c_{2,1} = c_{3,1} = \ldots = 0$. With three terms, one achieves

$$\frac{1}{c_{0,1}} - c_{1,1} = \frac{\varphi(\beta)}{\Phi(-\beta)} \tag{7-45}$$

$$\frac{1}{c_{0,1}^2} - 2\frac{c_{1,1}}{c_{0,1}} + 2c_{2,1} = \frac{\beta\varphi(\beta)}{\Phi(-\beta)} \tag{7-46}$$

$$\frac{1}{c_{0,1}^3} - 3\frac{c_{1,1}}{c_{0,1}^2} + 6\frac{c_{2,1}}{c_{0,1}} = \frac{(\beta^2 - 1)\varphi(\beta)}{\Phi(-\beta)}. \tag{7-47}$$

The three equations can be summarized to a cubic equation with at least one positive solution. The solution of $c_{0,1}$ less than the value assessed with the following formulae should be taken:

$$c_{0,1} = \frac{\Phi(-\beta)}{\varphi(\beta)}\left(\frac{1}{1+\sqrt{1-\beta\Phi(-\beta)/\varphi(\beta)}}\right) \tag{7-48}$$

Based on the cut off of the terms, different approximation formulations can be derived. For one term with negative curvature, the coefficient becomes

$$c_{0,2} = \frac{\Phi(\beta)}{\varphi(\beta)} \tag{7-49}$$

and $c_{1,2} = c_{2,2} = \ldots = 0$. If two terms are chosen, then the coefficients can be computed as

$$c_{0,2} = \frac{\Phi(\beta)}{\varphi(\beta)}\left(\frac{1}{1+\sqrt{1+\beta\Phi(\beta)/\varphi(\beta)}}\right) \tag{7-50}$$

$$c_{1,2} = \frac{-\varphi(\beta)}{\Phi(\beta)}\sqrt{1+\frac{\beta\Phi(\beta)}{\varphi(\beta)}} \tag{7-51}$$

and $c_{2,2} = c_{3,2} = \ldots = 0$. With three terms, one achieves

$$\frac{1}{c_{0,2}} + c_{1,2} = \frac{\varphi(\beta)}{\Phi(\beta)} \tag{7-52}$$

$$\frac{1}{c_{0,2}^2} + 2\frac{c_{1,2}}{c_{0,2}} + 2c_{2,2} = -\frac{\beta\varphi(\beta)}{\Phi(\beta)} \tag{7-53}$$

$$\frac{1}{c_{0,2}^3} + 3\frac{c_{1,2}}{c_{0,2}^2} + 6\frac{c_{2,2}}{c_{0,2}} = \frac{(\beta^2 - 1)\varphi(\beta)}{\Phi(-\beta)} \tag{7-54}$$

Again, the three equations can be summarized in to a cubic equation with at least one positive solution. The solution of $c_{0,2}$ less than the value assessed with the following formulae should be taken.

$$c_{0,2} = \frac{\Phi(\beta)}{\varphi(\beta)} \left(\frac{1}{1 + \sqrt{1 - \beta\Phi(\beta)/\varphi(\beta)}} \right) \tag{7-55}$$

If the d-terms are approximated, one achieves for a $d_{0,1} = 2c_{0,1}$, $d_{0,2} = 2c_{0,2}$ $d_{1,1} = d_{2,1} = \ldots = 0$ and $d_{1,2} = d_{2,2} = \ldots = 0$ one-term formulation.

7.2.3.4 Method by Cai and Elishakoff

Cai and Elishakoff (1994) also try to improve Breitung's method by extending the original formulae into a Taylor theorem. The application is quite simple:

$$P_f = \Phi(\beta) + \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\beta^2}{2} \right) (D_1 + D_2 + D_3 + \ldots) . \tag{7-56}$$

The single elements of the Taylor theorem are

$$D_1 = \sum_j \lambda_j \tag{7-57}$$

$$D_2 = -\frac{1}{2}\beta \left(3 \cdot \sum_j \lambda_j^2 + \sum_{j \neq k} \lambda_j \lambda_k \right) \tag{7-58}$$

$$D_3 = \frac{1}{6}(\beta^2 - 1) \left(15 \cdot \sum_j \lambda_j^3 + 9 \cdot \sum_{j \neq k} \lambda_j^2 \lambda_k + \sum_{j \neq k \neq l} \lambda_j \lambda_k \lambda_l \right) . \tag{7-59}$$

The basis for the computation of the elements is again the principal curvatures. However, these values have already been computed, if Breitung's formulae were employed:

$$a_j = -2\lambda_j . \tag{7-60}$$

7.2.3.5 Further developments

The latest SORM method was introduced by Zhao and Ono (1999a, b) and by Polidori et al. (1999). The first method is based on the inverse fast Fourier transformation (IFFT). These methods will not be explained here. However, it should be noted that Zhao and Ono (1999a) not only give a summary about the conditions under which the different introduced SORM methods perform well, but also impose recommendations on when to use FORM, when to use SORM methods, and when to use their own method (Zhao and Ono 1999c). However, Breitung (2002) has criticized some assumptions of the method by Zhao and Ono.

7.2.4 Hypersphere Division Method

Additional to the optimization procedure using Cartesian coordinates, the search for the minimum distance can also be carried out in spherical coordinates (see Fig. 7-7). After transformation into spherical coordinates, the angle and radius of a search vector are systematically adapted to find the minimum distance.

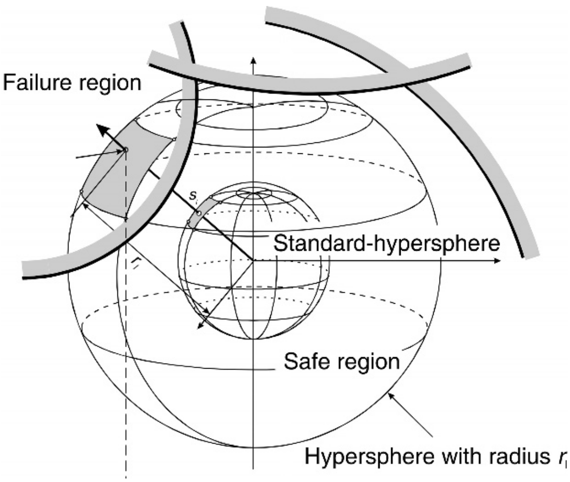


Fig. 7-7. Hypersphere division method

7.2.5 Response Surface Method

The probabilistic methods introduced so far required an analytically closed known limit state function $g(\mathbf{X})$. In other words, the limit state function should be one formula. However, in many cases, such as the computation of the ultimate load of an arch bridge, this requirement cannot be fulfilled. For example, consider a finite element program code. In those cases, the mathematical procedure has to be approximated by a surrogate, to reach to an acceptable computation when the aforementioned techniques like FORM and SORM are used. A procedure to develop such a surrogate is the response surface methodology. In this methodology, a simplified more dimensional function is computed based on some sample results of the extensive mathematical procedure originally established. The concept is easily understood when the complicated mathematical computations are substituted by some laboratory or field tests. There also, no function is known, but should be introduced. Based on the test results, a function will be introduced. Some general works about the concept can be found in Box and Draper (1987) and applications in structural engineering are mentioned in Bucher and Bourgund (1990) and Rajashekhar and Ellingwood (1993).

The concept can be described as follows. A certain function with the input variables **X** and some functional constants **K** is given with

$$g = f(\mathbf{X}, \mathbf{K}) \tag{7-61}$$

but can only be pointwise solved. Therefore, an approximation function should be developed

$$\tilde{g} = f(\mathbf{X}, \mathbf{K})\,. \tag{7-62}$$

There are many different types of mathematical approximation functions. Probably the most applied methods are quadratic functions, either

$$\tilde{g} = a + \sum_{i=1}^{n} b_i \cdot x_i + \sum_{i=1}^{n} c_i \cdot x_i^2 \tag{7-63}$$

or with mixed terms

$$\tilde{g} = A + \mathbf{X}^T \cdot \mathbf{B} + \mathbf{X}^T \cdot \mathbf{C} \cdot \mathbf{X} \tag{7-64}$$

with A, **B** and **C** as constants.

$$\mathbf{B} = \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & & & \\ \vdots & & & \\ C_{n1} & & & C_{nn} \end{pmatrix}. \tag{7-65}$$

These constants can be computed based on the pointwise solutions. Depending on the number of pointwise solutions, there is an under determined number of solutions situation, an exact number, or an over determined number of solutions to compute **K**. If an over determined number of solutions exist, then some minimum error methods should be applied. For the exact number of the solutions, the degrees of freedom of functions depending on the type of the function are shown in Table 7-2. Shapes of solutions points are shown in Fig. 7-8.

Table 7-2. Degrees of freedom for certain response surface functions (Weiland 2003)

Approximation function $\tilde{g}(\boldsymbol{x})$		Degrees of freedom
Linear regression	$\tilde{g}(\boldsymbol{x}) = a + \sum_{i=1}^{n} b_i \cdot x_i$	$n+1$
Quadratic function including mixed terms	$\tilde{g}(\boldsymbol{x}) = a + \sum_{i=1}^{n} b_i \cdot x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \cdot x_i \cdot x_j$	$\frac{1}{2} \cdot (n+1) \cdot (n+2)$
Quadratic function without mixed terms	$g(\boldsymbol{x}) = a + \sum_{i=1}^{n} b_i \cdot x_i + \sum_{i=1}^{n} x_i^2$	$2n+1$
Polynomial third order with mixed terms	…	$\frac{1}{2} \cdot (2+3n+3n^2)$
Polynomial third order without mixed terms	…	$3n+1$
Polynomial fourth order with mixed terms	…	$\frac{1}{2} \cdot (2+3n+5n^2)$
Polynomial fourth order without mixed terms	…	$5n+1$

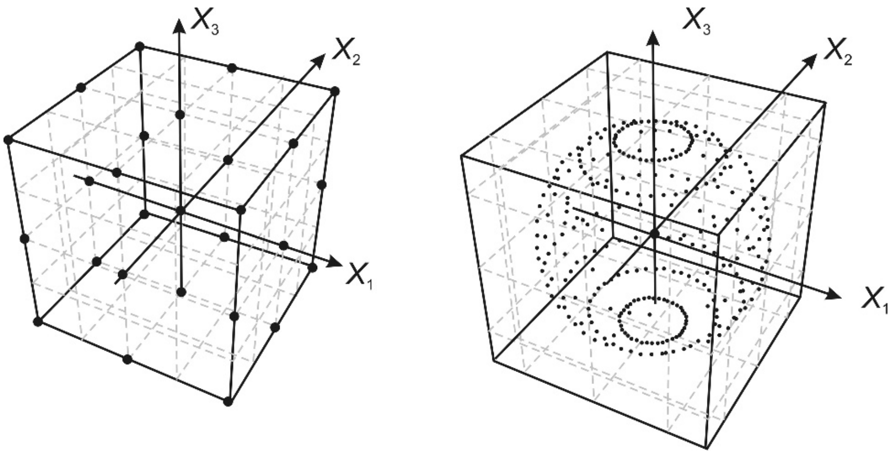


Fig. 7-8. Two examples of chosen solution points for three variables (Weiland 2003)

The approximated response surface can then be updated after the next probabilistic computation. This means that the response surface mainly acts as a local approximation and does not give a good approximation over the entire range of the original function. However, that is not required for FORM/SORM computations:

$$x_{D=m}^{(k+1)} = x_m^{(k)} + (x_D^k - x_m^{(k)}) \frac{g(x_m^{(k)})}{(g(x_m^{(k)}) - g(x_D^k))} \tag{7-66}$$

with x_m as centre point, x_D as design point based on a FORM computation using the response surface, and k as iteration counter. The iteration scheme is shown in Fig. 7-9.

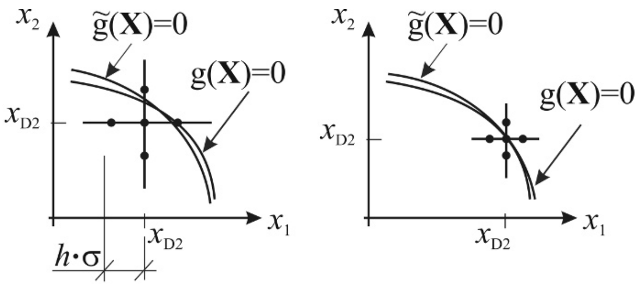


Fig. 7-9. Iterative improvement of the response surface (Klingmüller and Bourgund 1992)

The major advantage of this approximation is a simple application. The schema can easily be extended to existing finite element programs or other numerical tools. The computation is easily understandable and the number of computations is low.

The major disadvantage is a limited capability to find the extreme value of complicated functions. The number of iterations also depends on the number of random variables. Therefore, in high-dimensional cases, the response surface method may perhaps cause heavy computations. Furthermore, the approximation method only uses data points from one iteration cycle. However, it may perhaps be useful to keep the data for further investigations. There exist external programs that can carry out response surface computations afterwards by using all available data.

Since the limitations of the response surface method are known, in the last few years many new methods have been developed (Roos and Bayer 2008, Ross and Bucher 2003). An adaptive response surface method has been introduced by Most (2008).

7.2.6 Monte Carlo Simulation

7.2.6.1 Crude Monte Carlo Simulation

In contrast to the FORM and SORM methods, the Monte Carlo Simulation is an integration procedure, not an extreme value computation. Therefore, some assumptions required for the FORM/SORM are not relevant for the Monte Carlo Simulation. Also, Monte Carlo Simulation is extremely easy to program and to apply. However, if the probabilities of failure are extremely low and as required by codes, then Monte Carlo Simulation will require extensive computation power.

The general idea of Monte Carlo Simulation is, as the name already indicates, the application of pure random numbers into a computation flow. In its simplest description, Monte Carlo Simulation is an extensive version of trial and error. The only assumption for this technique is some quality requirements for random numbers. Since computers cannot provide real random numbers, they produce pseudo-random numbers based on purely deterministic causal computations; the period of the numbers should be big enough so that random numbers are not repeated in the Monte Carlo Simulation. There are many programs available to provide high-quality random numbers (NR 1992). The Monte Carlo Simulation is then

$$\int f \; dV \approx V\left\langle f \right\rangle \pm V\sqrt{\frac{\left\langle f^2 \right\rangle - \left\langle f \right\rangle^2}{N}}, \tag{7-68}$$

where V represents the volume, $V\left\langle f \right\rangle$ stands for the mean value of the function f over the sample size N, and the ±-term gives more or less an one standard deviation error estimator. The further functions are

$$\left\langle f \right\rangle \equiv \frac{1}{N}\sum_{i=1}^{N} f(x_i) \qquad \left\langle f^2 \right\rangle \equiv \frac{1}{N}\sum_{i=1}^{N} f^2(x_i). \tag{7-69}$$

The major advantage of the Monte Carlo Simulation concerning the dimensions is the fact that the statistical error is independent from the number of dimensions. For Monte Carlo Simulation, it does not matter if there is 1 or 50 random input variables, the statistical error will remain the same. This is completely different for Simpson's rule applied for integration, where the required computation grows exponentially with the number of dimensions.

However, the required sample size has a major influence on the error size in Monte Carlo Simulation. The next equation is an example to evaluate the required sample size n_{req} (Flederer 2001)

$$n_{req} = \frac{1}{1-P_\varepsilon} \cdot \frac{(1-P_f)}{P_f \cdot \varepsilon^2} \tag{7-70}$$

with P_f as probability of failure, P_ε as level of significance, and ε as statistical error. From the equation, it becomes understandable that with low probabilities of failure and low statistical error, high sample sizes are required. Macke (2000) has given a good example, where the probability of failure was 10^{-6} and the required statistical error was less than 50%. The required sample size was 4×10^6. If the statistical error should be less than 10%, then more than 10^8 samples are required. Based on these properties, Monte Carlo Simulation is a good method for high probabilities of failure and for high dimensions. If lower probabilities were to be investigated with Monte Carlo Simulation, the so-called variance reduction techniques should be applied. This is mainly done after a preliminary FORM/SORM computation.

7.2.6.2 Variance-reduced Monte Carlo Simulation

Since a great variety of variance reduction techniques exist, here only a few will be mentioned. Probably the most applied technique is importance sampling. Here, the original probability integral

$$P_f = \int \ldots \int_{g<0} f_x(\mathbf{x}) \mathrm{d}\mathbf{x} \tag{7-71}$$

will be transferred using an indicator or a weighting function to

$$P_f = \int \ldots \int_{\text{all}} I(x) f_x(\mathbf{x}) \mathrm{d}\mathbf{x} \,. \tag{7-72}$$

The weighting function is defined as

$$I(\mathbf{x}) = \begin{cases} 1 & g(\mathbf{x}) < 0 \\ 0 & g(\mathbf{x}) \geq 0 \end{cases} \tag{7-73}$$

This permits a transformation of the random distributions using a chosen distribution function $h_v(\mathbf{v})$:

$$P_f = \int \ldots \int_{\text{all}} I(\mathbf{v}) \frac{f_x(\mathbf{v})}{h_v(\mathbf{v})} h_v(\mathbf{v}) \mathrm{d}\mathbf{v} \,. \tag{7-74}$$

This function can be unbiasedly estimated with

$$\hat{P}_f = \frac{1}{m_c} \sum_{n=1}^{m_c} I(\mathbf{v}_n) \frac{f_x(\mathbf{v}_n)}{h_v(\mathbf{v}_n)} . \tag{7-75}$$

Furthermore, the variance can be computed as

$$\mathrm{Var}[\hat{P}_f] = \frac{1}{m_c - 1} \left[\frac{1}{m_c} \sum_{n=1}^{m_c} I(\mathbf{v}_n) \left(\frac{f_x(\mathbf{v}_n)}{h_v(\mathbf{v}_n)} \right)^2 - \hat{P}_f^2 \right] . \tag{7-76}$$

The major task in applying importance sampling is the search for a proper distribution function $h_v(\mathbf{v})$. If some prior information is available, for example by FORM/SORM computation, then the distribution function $h_v(\mathbf{v})$ can be selected very efficiently, yielding to an impressive drop of computational effort in the Monte Carlo Simulation. In general, importance sampling can be understood as a transformation of the random points toward interesting regions in the sampling space (Maes et al. 1993, Song 1997, and Ibrahim 1991).

The concept can be even further extended by updating the distribution function $h_v(\mathbf{v})$ after every sample step. This technique is called adaptive sampling (Bucher 1988 and Mori and Ellingwood 1993).

7.2.6.3 Quasi-random numbers

Another interesting technique is the application of quasi-random numbers instead of pseudo-random numbers. Quasi-random numbers are not random at all. They are constructed to fill a multidimensional space in a most efficient way. Also, they can be understood as a technique standing between the classical Simpson's rule for integration and the crude Monte Carlo Simulation. Since the numbers are deterministic, the computation error becomes related to the dimensions.

However, the major advantage of the application of quasi-random numbers with Monte Carlo Simulation is the fact that they can be applied to many finished programs. So, if Monte Carlo Simulation was chosen for a certain project and it turns out after some sample computations that the computation time is unacceptable, but the programming cannot be changed anymore, then perhaps quasi-random numbers can be produced externally and can then given to the program. This technique was applied in Flederer (2001). More details about possible application can be found in Curbach et al. (2002).

7.2.7 Combination of Safety Indexes

Whereas for Monte Carlo Simulation the number of limit state functions is irrelevant for the FORM and SORM methods, perhaps different limit state functions were considered separately and have to be merged into one single probability of failure or safety index. This can be seen at an arch bridge, where several different point of failure can be identified. The question that follows is whether these points are correlated or not.

If the different limit state functions are uncorrelated, then system probability of failure can be computed as

$$P_f = 1 - \prod_{j=1}^{n} (1 - P_{fj}) . \tag{7-77}$$

If the single probabilities of failure are rather small, the values can be added instead of multiplying

$$P_f \approx \sum_{j=1}^{n} P_{fj} . \tag{7-78}$$

The error term is then not higher than

$$E(P_f) \leq \frac{1}{2} \left(\sum_{j=1}^{n} P_{fj} \right)^2 . \tag{7-79}$$

If the probabilities of failure are expressed as safety indexes, the formulas become

$$\beta_{sys} = -\Phi^{-1} \left(1 - \prod_{j=1}^{n} \Phi(\beta_j) \right) \approx -\Phi^{-1} \left(\sum_{j=1}^{n} \Phi(-\beta_j) \right) . \tag{7-80}$$

If the noncorrelation of the limit states does not hold true, then the correlations have to be considered and a multidimensional normal distribution can be applied. The correlations are expressed as

$$\rho_{jk} = \alpha_j^T \alpha_k = \alpha_{j1}\alpha_{k1} + \alpha_{j2}\alpha_{k2} + ... + \alpha_{jm}\alpha_{km} \tag{7-81}$$

and

$$\alpha_j^T = (\alpha_{j1} + \alpha_{j2} + ... + \alpha_{jm}) . \tag{7-82}$$

With α_j^T are the weighting factors of the m random variables from the j^{th} limit state function giving the safety index β_{j}. For the correlation matrix of the different limit state functions, one gets

$$R = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & & \vdots \\ \vdots & & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{pmatrix} \tag{7-83}$$

and a vector for the safety indexes

$$\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix}. \tag{7-84}$$

The system probability of failure is obtained by transformation in the standard normal space and linearization of the limit state functions:

$$P_f = 1 - P_s \tag{7-85}$$

$$= 1 - P\left(\bigcap_{j=1}^{n}(g_j(X) \geq 0)\right) \tag{7-86}$$

$$= 1 - P\left(\bigcap_{j=1}^{n}(h_j(Y) \geq 0)\right) \tag{7-87}$$

$$\approx 1 - P\left(\bigcap_{j=1}^{n}(l_j(Y) \geq 0)\right) \tag{7-88}$$

$$= 1 - P\left(\bigcap_{j=1}^{n}(-Z_j^* \leq \beta_j)\right) \tag{7-89}$$

$$= 1 - \Phi_n(\beta, \mathbf{R}), \tag{7-90}$$

with $\Phi_n(\beta, \mathbf{R})$ as standardized n-dimensional normal distribution.

$$\Phi_n(\beta, \mathbf{R}) = \frac{1}{(2\pi)^{n/2}|R|^{1/2}} \int_{-\infty}^{\beta_n} \cdots \int_{-\infty}^{\beta_1} \exp\left(-\frac{1}{2}\psi^T \mathbf{R}^{-1}\mathbf{y}\right) \mathrm{d}y_1, \mathrm{d}y_2, \cdots \mathrm{d}y_n. \tag{7-91}$$

Several different programs can be found for the evaluation of this function (Schervish 1984, Genz 1992, Drezner 1992, and Yuan and Pandey 2006).

To simplify the computation, it is often assumed that the correlations are equal between different limit state functions. The correlation matrix becomes

$$R = \begin{pmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & & \vdots \\ \vdots & & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{pmatrix} \tag{7-92}$$

and the multidimensional standard normal distribution changes to

$$\Phi_n(\beta, \mathbf{R}) = \int_{-\infty}^{+\infty} \varphi(x) \prod_{i=1}^{n} \Phi\left(\frac{\beta_i + \sqrt{\rho} \cdot x}{\sqrt{1-\rho}} \right) \mathrm{d}x \,. \tag{7-93}$$

Based on this idea of a mean correlation matrix, the coefficients of the correlation matrix can be understood as products of single elements, like

$$R = \begin{pmatrix} 1 & \lambda_1 \cdot \lambda_2 & \cdots & \lambda_1 \cdot \lambda_n \\ \lambda_2 \cdot \lambda_1 & 1 & & \vdots \\ \vdots & & \ddots & \vdots \\ \lambda_n \cdot \lambda_1 & \lambda_n \cdot \lambda_2 & \cdots & 1 \end{pmatrix}, \ |\lambda_{\mathrm{i}}| < 1, \ |\lambda_{\mathrm{i}}| < 1, \ i, j = 1, 2, \ldots, n \tag{7-94}$$

and the multidimensional standard normal distribution again changes

$$\Phi_n(\beta, \mathbf{R}) = \int_{-\infty}^{+\infty} \varphi(x) \prod_{i=1}^{n} \Phi\left(\frac{\beta_i + \lambda_i \cdot x}{\sqrt{1-\lambda_i^2}} \right) \mathrm{d}x \,. \tag{7-95}$$

Suggestions of lower bounds for the correlations values are given as

$$\lambda_j \cdot \lambda_k \le \rho_{jk} \quad j \ne k \tag{7-96}$$

and for the upper bound as

$$\lambda_j \cdot \lambda_k \ge \rho_{jk} \quad j \ne k \tag{7-97}$$

and

$$-1 \le \lambda_j, \lambda_k \le 1 \tag{7-98}$$

should remain valid. An optimal solution would be achieved when

$$\lambda_j \cdot \lambda_k = \rho_{jk} . \tag{7-99}$$

That will only be possible in some rare cases. However, if the ρ_{jk} has approximately the same size and is positive, then the following recommendation for lower bounds

$$\lambda_j = \sqrt{\max_j \{\rho_{jk}\}} \quad j \neq k \tag{7-100}$$

and upper bounds

$$\lambda_j = \sqrt{\min_j \{\rho_{jk}\}} \quad j \neq k . \tag{7-101}$$

can be given.

If the contribution of the limit state functions to the system probability of failure can be roughly estimated, then the three limit state functions with the highest probability of failure or the lowest safety index can be chosen, and the following computation can be carried out:

$$\lambda_1 \cdot \lambda_2 = \rho_{12}, \quad \lambda_2 \cdot \lambda_3 = \rho_{23}, \quad \lambda_1 \cdot \lambda_3 = \rho_{13} \tag{7-102}$$

$$\lambda_1 = \sqrt{\frac{\rho_{12} \cdot \rho_{13}}{\rho_{23}}}, \quad \lambda_2 = \sqrt{\frac{\rho_{21} \cdot \rho_{23}}{\rho_{13}}}, \quad \lambda_3 = \sqrt{\frac{\rho_{31} \cdot \rho_{32}}{\rho_{12}}} \quad . \tag{7-103}$$

For the remaining values, an upper bound

$$\lambda_j = \min_{k \leq j+1} \left\{ \frac{\rho_{jk}}{\lambda_k} \right\} \quad j = 4, 5... \tag{7-104}$$

and a lower bound can be estimated

$$\lambda_j = \max_{k \leq j+1} \left\{ \frac{\rho_{jk}}{\lambda_k} \right\} \quad j = 4, 5... \quad . \tag{7-105}$$

Unfortunately, the requirement

$$-1 \leq \lambda_j, \ \lambda_k \leq 1 \tag{7-106}$$

can sometimes cause numerical problems.

For series system, further bounds can be given. In serious systems, the probability of failure increases by decreasing the correlation between the different elements or limit state functions. This can be understood as additionally random effects. The boundaries for a series system can be written as

$$\max_j P_{fj} \le P_f \le 1 - \prod_{j=1}^{n}(1 - P_{fj}) < \sum_{j=1}^{n} P_{fj} \tag{7-107}$$

or, in terms of the safety index

$$\min_j \beta_j \ge \beta_{sys} \ge -\Phi^{-1}\left(1 - \prod_{j=1}^{n}\Phi(\beta_j)\right) > -\Phi^{-1}\left(\sum_{j=1}^{n}\Phi(-\beta_j)\right). \tag{7-108}$$

Based on general additive theorems for probabilities, more precise bounds can be given:

$$P_f \le \min\begin{cases} 1 \\ \sum_{j=1}^{n} P(F_j) - \sum_{j=2}^{n} \max_{k<j} P(F_j \cap F_k) \end{cases} \tag{7-109}$$

$$P_f \ge P(F_1) + \sum_{j=2}^{n} \max\begin{cases} 0 \\ P(F_j) - \sum_{k=1}^{j-1} P(F_j \cap F_k) \end{cases}. \tag{7-110}$$

The average volume of two probabilities of failure can be computed as

$$P(F_j \cap F_k) \approx \Phi_2(-\beta_j, -\beta_k; \rho_{jk}). \tag{7-111}$$

Using this for the above given boundaries, one achieves

$$P_f \le \min\begin{cases} 1 \\ \sum_{j=1}^{n} \Phi(-\beta_j) - \sum_{j=2}^{n} \max_{k<j} \Phi_2(-\beta_j, -\beta_k; \rho_{jk}) \end{cases} \tag{7-112}$$

$$P_f \ge \Phi(-\beta_1) + \sum_{j=2}^{n} \max\begin{cases} 0 \\ \Phi(-\beta_j) - \sum_{k=1}^{j-1} \Phi_2(-\beta_j, -\beta_k; \rho_{jk}) \end{cases}. \tag{7-113}$$

The two-dimensional standard normal distribution can be approximated with

$$\Phi_2(x_2;x_2;\rho) = \int_{-\infty}^{+\infty} \Phi\left(\frac{x_1 + \lambda_1 \cdot w}{\sqrt{1-\lambda_1^2}}\right) \cdot \Phi\left(\frac{x_1 + \lambda_1 \cdot w}{\sqrt{1-\lambda_1^2}}\right) \cdot \varphi(w)\ dw \tag{7-114}$$

using

$$\lambda_1 = \lambda_2 = \sqrt{\rho},\ \rho > 0 \tag{7-115}$$
$$\lambda_1 = \sqrt{-\rho},\ \lambda_2 = -\sqrt{-\rho},\ \rho < 0\,.$$

The introduced methods have been programmed into FORTRAN 77 routines and are available free of charge to the reader of this book. Please simply contact the authors.

Arch bridges are usually considered to be a serious system: if one part fails, the entire bridge will collapse. However, some parts of bridge behave like parallel systems: one part will collapse and other parts will take more loads. Further works about the estimation of probabilities of failure for such types of systems can be found in Rackwitz and Hohenbichler (1981) or Gollwitzer and Rackwitz (1990), as seen in Fig. 7-10.

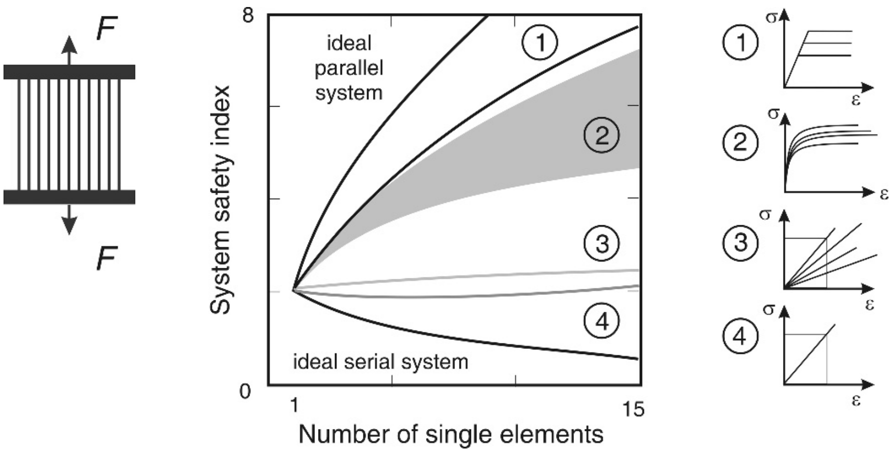


Fig. 7-10. Relationship between system safety index and material properties shown for a Daniel system

7.2.8 Limitation of the Presented Methods

The presented methods so far have described uncertainties of materials and loads by random distribution functions. They have not considered any correlations between the random variables that increase the numerical work. For the interested reader, the Rosenblatt transformation or the NATAF transformations are dealt with in Melchers (1999) or Liu and Der Kiureghian (1986). Copulas are an additional technique to transform the correlated random variables into noncorrelated variables.

Furthermore, these random distribution functions are usually kept constant over different distances or volumes. In contrast, it is well-known that, for example, material properties might not have a full correlation over a certain distance or volume. This correlation change over distance may be described by random fields (Vanmarcke 1983). In past years, random fields are increasingly applied in structural safety investigations to establish stochastic finite elements (Der Kiureghian and Ke 1988, Ghanem and Spanos 1991, and Pukl et al. 2006). The latest advances were shown by Bayer and Ross (2008).

Furthermore, not all presented techniques perform well under all conditions. Figures 7-11 and 7-12 give a good overview about the application conditions of the probabilistic techniques.

It is not intended by the authors to give here a full summary about current state of knowledge in the field of structural safety. A state-of-the-art report for computational stochastic mechanics was given by Berman et al. (1997), however, the latest developments should be considered. In contrast, the introduced methods can be easily programmed and applied to arch bridge problems by the reader. For more advanced studies, some commercial programs or programs from research institutes can be used.

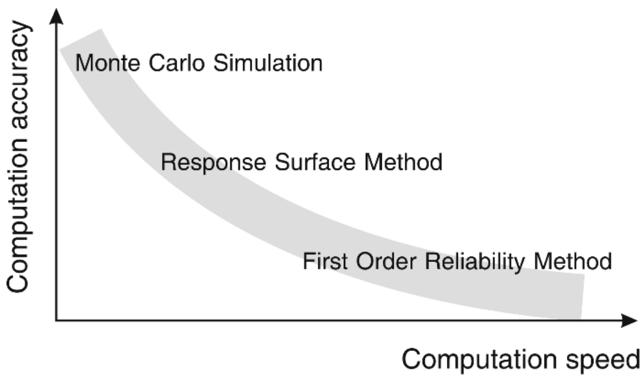


Fig. 7-11. Performance of methods for stochastic structural analysis (Bucher et al. 2000)

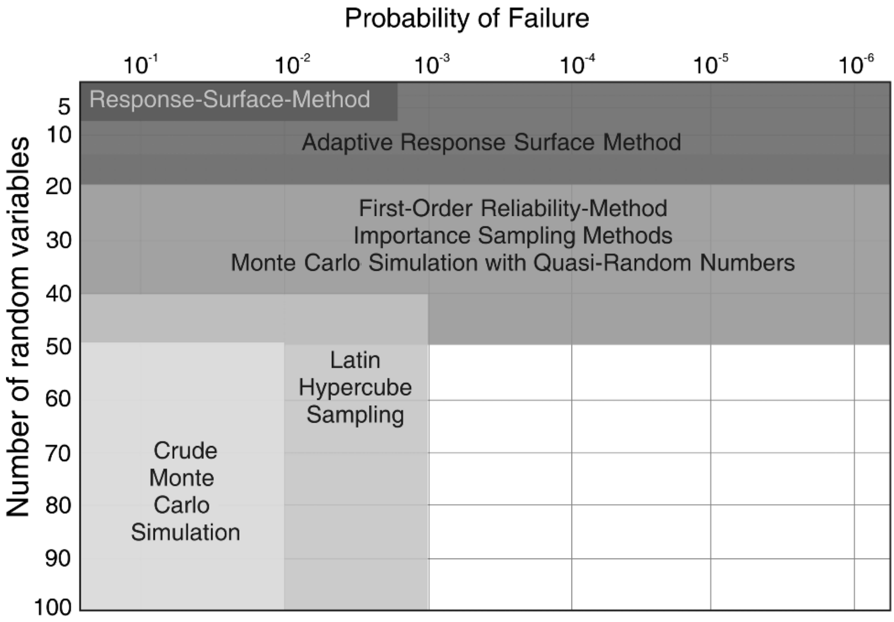


Fig. 7-12. Applicability of certain stochastic techniques subject to the number of random variables and the estimated probability of failure (Bayer 2008)

7.2.9 Commercial Programs

Although many universities have developed programs for the computation of probabilities of failure, in many cases these programs lack a sufficient manual, a graphical user interface, or simply an easy handling. Therefore, in many cases, commercial probabilistic programs were developed.

Currently, the following programs are known to the authors, however this list is subject to change: UNIPASS (Lin and Khalessi 2006), ProFES (Wu et al. 2006), Proban (Tvedt 2006), PHIMECA (Lemaire and Pendola 2006), PERMAS-RA/STRUREL (Gollwitzer et al. 2006), NESSUS (Thacker et al. 2006), COSSAN (Schueller and Pradlwarter 2006), CalRel/ FERUM/ OpenSees (Der Kiureghian et al. 2006), ANSYS PDS und DesignXplorer (Reh et al. 2006), ATENA/SARA/FREET (Pukl et al. 2006), VaP (Petschacher 1994), OptiSlang (Schlegel and Will 2007), RELSYS (Estes and Frangopol 1998), and the probabilistic toolbox ProBox (Schweckendiek and Courage 2006). Many of these programs can be downloaded free of charge for test runs (Table 7-3).

As mentioned above, parallel to the listed commercial programs, many further stand alone programs were or are under development in companies or at universities. Therefore, Epstein et al. (2008) suggested some general requirements and structures for probabilistic computer programs. The adaptation of these rules will ease the application of such programs and will extend the user group.

Even without commercial programs, simple FORM or Monte Carlo Simulations can be carried out with standard spreadsheet software. Including an optimization tool like the solver in EXCEL, it is possible to compute the safety index. An example of such an application can be found in Low and Teh (2000).

Table 7-3. List of different probabilistic programs currently available

Program	University	Homepage
VAP	ETH Zurich	http://www.ibk.baum.ethz.ch/proserv/vap.html
CALREL	University of California, Berkeley	http://www.ce.berkeley.edu
FERUM	University of California, Berkeley	http://www.ce.berkeley.edu/~haukaas/FERUM/ferum.html
NESSUS	Southwest Research Institute, San Antonia	http://www.nessus.swri.org/
PERMAS	INTES GmbH, Stuttgart	http://www.intes.de
SLANG	Bauhaus University, Weimar	http://www.uni-weimar.de/Bauing/ism/Slang
ISPUD	University Innsbruck	http://www.uibk.ac.at/c/c8/c810
COSSAN	University Innsbruck	http://www.uibk.ac.at/c/c8/c810
PROBAN	Det Norske Veritas Software	http://www.dnv.com
STRUREL	RCP GmbH	http://www.strurel.de
ANSYS	ANSYS Inc.	http://www.ansys.com
SARA FREET	University Bruno, Cervenka Consulting	http://www.cervenka.cz
RACKV	University of Natural Resources and Applied Life Sciences, Vienna	

7.2.10 Goal Values of Safety Indexes

If a safety index or a probability of failure is used as a measure for safety of the structure, they have to be compared to a certain proof or goal value. Many references have published such goal probabilities of failure and examples are shown in Tables 7-4 to 7-10. However, most of the publications show nearly the same values; for new structures, maximum probability of failure is in the range of 10^{-6} per year or a minimum safety index of 3.8 per year. For existing structures, usually less stringent requirements are common. For example, a decrease of the safety index by 0.5 (Diamantidis et al. 2007). Furthermore, the probability of failure or the safety index can be adapted to more special conditions.

Table 7-4. Goal probability of failure per year according to the CEB (1976)

Average number of people endangered	Economical consequences low	average	high
Low (< 0.1)	10^{-3}	10^{-4}	10^{-5}
Average	10^{-4}	10^{-5}	10^{-6}
High (> 10)	10^{-5}	10^{-6}	10^{-7}

Table 7-5. Goal probability of failures in some Scandinavian countries (Spaethe 1992)

Safety class	Failure consequences	Probability of failure for the limit state of ultimate load per year
Low	Low personal injuries Insignificant economical consequences	$1.0 \times \cdot 10^{-4}$
Normal	Some personal injuries Considerable economical consequences	1.0×10^{-5}
High	Considerable personal injuries Very high economical consequences	1.0×10^{-6}

Table 7-6. Goal probability of failures in the former East Germany (Franz et al. 1991)

Reliability class	Consequences	Probability of failure
I	Very high danger to the public Very high economical consequences Disaster	1.0×10^{-7}
II	High danger to the public High economical consequences High cultural losses	1.0×10^{-6}
III	Danger to some persons Economical consequences	1.0×10^{-5}

Reliability class	Consequences	Probability of failure
IV	Low danger to persons Low economical consequences	1.0×10^{-4}
V	Very low danger to persons Very low economical consequences	7.0×10^{-4}

Table 7-7. Goal probability of failures according to the GruSiBau (1981)

Safety class	Possible consequences of failure		Type of limit state	
	Limit state of ultimate load bearing	Limit state of serviceability	Ultimate load	Serviceability
1	No danger to humans and no economical consequences	Low economical consequences and low usage limitation	1.34×10^{-5}	6.21×10^{-3}
2	Some danger to humans and considerable economical consequences	Considerable economical consequences and strong limitation of further usage	1.30×10^{-6}	1.35×10^{-3}
3	High importance of the structure to the public	High economical consequences and high restriction to future usage	1.00×10^{-7}	2.33×10^{-4}

Table 7-8. Goal probability of failure according to the DIN 1055-100 (1999) and the Eurocode 1 (1994)

Limit state	Probability of failure	
	Lifetime	Per year
Ultimate load	7.24×10^{-5}	1.30×10^{-6}
Serviceability	6.68×10^{-2}	1.35×10^{-5}

Table 7-9. Goal safety indices according to ISO/CD 13822 (1999)

Limit state	Safety index
Serviceability	
Reversible	0.0
Irreversible	1.5
Fatigue	
Testable	2.3
Not testable	3.1
Ultimate load	
Very low consequences	2.3
Low consequences	3.1
Common consequences	3.8
High consequences	4.3

Table 7-10. Goal safety indices according to the JCSS Modelcode (2004)

Costs for safety measures	Low failure consequences	Average failure consequences	High failure consequences
Low	3.1	3.3	3.7
Average	3.7	4.2	4.4
High	4.2	4.4	4.7

The Eurocode also permits an adaptation of the safety index to some consequence classes in terms of failure consequence classes (CC) as shown in Table 7-11. Such consequence classes can then be related to some reliability classes (RC) listed in Tables 7-12 and 7-13.

Table 7-11. Graduation of failure CCs according to the Eurocode 1 (1994)

Failure CCs	Consequences	Examples
CC 3	High consequences to humans, the economy, social systems and the environment	Stands, public buildings, for example concert halls
CC 2	Average consequences to humans, the economy, social systems and the environment	Dwelling and office buildings, public buildings such as offices
CC 1	Low consequences to humans, the economy, social systems, and the environment	Agricultural structures or structures without regular persons' residence, for example barns, conservatories

Table 7-12. Graduation of RCs according to the Eurocode 1 (1994)

RC	Safety index per year	Safety index for 50 years
RC 3	5.2	4.3
RC 2	4.7	3.8
RC 1	4.2	3.3

Table 7-13. Adaptation factor for the partial safety index subject to the RC (Eurocode 1 1994)

Adaptation for the partial safety factors	RC		
	RC1	RC2	RC3
K_{FI}	0.9	1.0	1.1

The Eurocode furthermore can consider different types of production control of the building material in terms of changes of partial safety factors of the material. Still, this is for new structures only. Therefore, some other recommendations focus on existing structures. For example, in Tables 7-14 and 7-15, some adaptation factors are given. Furthermore, Strauss and Bergmeister (2005) have also introduced some factors.

Table 7-14. Adaptation of the safety index according to the CAN/CSA-S6-88 Canadian Limit States Design Standard (taken from Casas et al. 2001 and COST 345 2004)

$\beta = 3.5 - (\Delta_E + \Delta_S + \Delta_I + \Delta_{PC}) \geq 2.0$	value
Correction factor for element failure	Δ_E
Abrupt failure without warning	0.0
Abrupt loss of bearing capacity without warning with remaining capacity	0.25
Grateful failure with warning	0.50
Correction factor for system failure	Δ_S
Failure of one single element causes system failure	0.00
Failure of one single element does not cause system failure	0.25
Failure of one single element causes local failure only	0.50
Correction factor for monitoring	Δ_I
Element is not controllable	–0.25
Element is controlled regularly	0.00
Critical elements are controlled more frequently	0.25
Correction factor for live load	Δ_{PC}
All types of traffic without special permission	0.00
All types of traffic with special permission	0.60

Table 7-15. Adaptation of the safety index according to Schueremans and Van Gemert (2001)

$\beta = \beta_T - (\Delta_S + \Delta_R + \Delta_P + \Delta_I) \geq 2.0$	value
Adjustment for system behaviour	Δ_S
Failure leads to collapse, likely to impact occupants	0.00
Failure is unlikely to lead to collapse, or unlikely to impact occupants	0.25
Failure is local only, very unlikely to impact on occupants	0.50
Adjustment for risk category	Δ_R
High number of occupants (n) exposed to failure (n = 100–1,000)	0.00

$\beta = \beta_T - (\Delta_S + \Delta_R + \Delta_P + \Delta_I) \geq 2.0$	value
Normal occupancy exposed to failure (n = 10–99)	0.25
Low occupancy exposed to failure (n = 0–9)	0.50
Adjustment for past performance:	Δ_P
No record of satisfactory past performance	0.00
Satisfactory past performance or dead load measured	0.25
Adjustment for inspection:	Δ_I
Component not inspect able	–0.25
Component regularly inspected	0.00
Critical component inspected by expert	0.25

This adapted safety index can then be used to provide alternative safety measures in the semiprobabilistic safety concept.

Further background information about the development of goal probabilities of failure or safety indexes can be found in Proske (2008b) discussing different risk parameters and the current developments.

7.3 Semiprobabilistic Safety Concept

7.3.1 Introduction

The probabilistic safety concept for structures has introduced the measure of probability of failure as a measure of reliability and safety. Nevertheless, under everyday conditions, this measure is not practical and instead, simpler types of proof of safety have to be used for structures. Therefore, the probabilistic safety concept has to be transformed into a semiprobabilistic safety concept, which means nothing else than developing substitutes for the probability of failure proof, which are easier to handle. Such substitutes are the safety factors, characteristic values, and the design values. They are developed on some basic simplification.

One major advantage for these elements is the long tradition of the application of safety factors. It has been estimated that the first application of a global safety factor goes back up to 300 B.C. by Philo from Byzantium (Shigley and Mischke 2001). He introduced the global safety factor in terms of

$$\gamma = \frac{\text{resistance}}{\text{load}} . \qquad (1\text{-}116)$$

Empirical geometrical rules remained valid over nearly the next two millenniums. Only in the last few centuries have the applications of safety factors become widespread. Over time, several different values were developed for different materials. In most cases, the values dropped significantly during the last century. As an example, in 1880 for brick masonry the safety factor of 10 was required, whereas only 10 years later the factor was chosen between 7 and 8. In the 20th century, the values ranged from factor 5 and 4, and now for the recalculation of historical structures with that material, factor 3 is chosen (Busch and Zumpe 1995, Schleicher 1949, Wenzel 1997, Mann 1987, and Tonon and Tonon 2006). This decline of safety factors could also be observed for other materials such as steel. The development of new materials especially led to more concerns about the safe application of those materials. The different developments for safety factors for materials led to the first efforts in the beginning of the 20th century to develop material-independent factors, as shown in Tables 7-16 and 7-17.

Table 7-16. Global safety factor according to Visodic (1948)

Safety factor	Knowledge of load	Knowledge of material	Knowledge of environment
1.2–1.5	Excellent	Excellent	Controlled
1.5–2.0	Good	Good	Constant
2.0–2.5	Good	Good	Normal
2.5–3.0	Average	Average	Normal
3.0–4.0	Average	Average	Normal
3.0–4.0	Low	Low	Unknown

Table 7-17. Global safety factor according to Norton (1996)

Safety factor	Knowledge of load	Knowledge of material	Knowledge of environment
1.3	Extremely well-known	Extremely well-known	Likewise tests
2	Good approximation	Good approximation	Controllable environment
3	Normal approximation	Normal approximation	Moderate
5	Guessing	Guessing	Extreme

As the tables show, a further decline of global safety factors seems to be limited; otherwise, the major requirement safety of structures might not be fulfilled anymore. Therefore, more advanced changes might be considered to meet the demanding requirements of economic and safe structures. Such a development would be special safety factors for the different columns in the table 7-16; for example, a safety factor for the load and a safety factor for the material. This is indeed the idea of the partial safety factor concept. It does not necessarily yield to lower safety factors, but it yields to a more homogenous level of safety.

The development of partial safety factors is strongly related to the development of the probabilistic safety concept. However, the practical application of partial safety factors took additional decades. First applications can be found in steel design, whereas a first example in the field of structural concrete could be ETV concrete—a concrete code in East Germany (ETV stands for Unified technical codes). ETV concrete was developed during the seventies of the 20th century and introduced in the beginning of the eighties. A comparison between ETV and the up-to-date German code DIN 1045-1 can be found in Wiese et al. (2005).

7.3.2 Partial Safety Factors

7.3.2.1 Introduction

The change from the global safety factor concept to the partial safety factor concept can be easily seen in the following equations. In general, the comparison of the resistance of a structure and the load or event remains

$$E_d \leq R_d \,. \tag{7-117}$$

But in contrast to the global safety factor format, where the safety factor can be separated like

$$E_d \leq R_d / \gamma_{Global} \,, \tag{7-118}$$

the safety factors accompany the parameters required for design. Then, the design load E_d is evaluated according to

$$E_d = \sum_{j \geq 1} \gamma_{G,j} \cdot G_{k,j} + \gamma_{Q,1} \cdot Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \cdot \psi_{0,i} \cdot Q_{k,i} \tag{7-119}$$

and the design resistance R_d is based on

$$R_d = R\left(\alpha \cdot \frac{f_{ck}}{\gamma_c}; \frac{f_{yk}}{\gamma_s}; \frac{f_{tk,cal}}{\gamma_s}; \frac{f_{p0,1k}}{\gamma_s}; \frac{f_{pk}}{\gamma_s} \right). \tag{7-120}$$

A list of material partial safety factors is given in Table 7-18.

Table 7-18. Material partial safety factors

Material	Code or reference	Limit state of ultimate load	Accidental load conditions
Concrete (up to C 50/60)	DIN 1045-1	1.50	1.30
Concrete higher than C 50/60	DIN 1045-1	1.5/ (1.1-f_{ck}/500)	1.3/ (1.1-f_{ck}/500)
Non-reinforced concrete	DIN 1045-1	1.80	1.55
Non-reinforced concrete	DAfSt (1996)	1.25	
Pre-cast concrete	DIN 1045-1	1.35	
Collateral evasion	DIN 1045-1	2.00	–
Reinforcement steel	DIN 1045-1	1.15	1.00
Pre-stressing steel	DIN 1045-1	1.15	1.00
Steel yield strength	EN 4	1.10	1.00
Steel tensile strength	EN 4	1.25	1.00
Steel tensile strength	EN 4	1.00	1.00
Wood	EN 5	1.30	1.00
Masonry (Category A)	EN 6	1.7 (I)/2.0 (II)	1.20
Masonry (Category B)	EN 6	2.2 (I)/2.5 (II)	1.50
Masonry (Category C)	EN 6	2.7 (I)/3.0 (II)	1.80
Masonry – steel	EN 6	1.50/2.20	
Masonry	DIN 1053-100	1.50–1.875	1.30–1.625
Wall anchorage (C. A-C)	Mann (1999)	2.50	1.20
Floatglas/Gussglas	BÜV (2001)	1.80	1.40
ESV-Glas	BÜV (2001)	1.50	1.30
Siliconglas	BÜV (2001)	5.00	2.50
Carbon fibre	Onken et al. (2002) and Bergmeister (2003)	1.20	
Carbon fibre		1.30[1]	
Carbon fibre cable		1.20[1]	
Carbon fibre glue		1.50[1]	
Cladding	DIN 18 516	2.00	
Aluminum yield strength	EN 9	1.10	
Aluminum tensile strength	EN 9	1.25	
Bamboo as building material	Bamboo (2005)	1.50	
Textile reinforced concrete	Own works	1.80[2]	
Concrete – multiaxial loading	Own works	1.35–1.70	
Granodiorit tensile strength	Own works	1.50–1.70	
Historical masonry arch bridges	UIC-Codex	2.00[3]	
Historical non-reinforced arch bridges	Bothe et al. (2004)	1.80[3]	

[1]Consider construction conditions (Bergmeister 2003).
[2]Recent researches indicate lower values.
[3]A partial safety factor for a system.

Unfortunately, the definition of the safety format for the resistance depends on the way of computing the forces in the structure. Eibl (1992) and Eibl and Schmidt-Hurtienne (1995) have pointed out the limitation of the partial safety factor concept in nonlinear force calculations. If the forces are computed in a nonlinear procedure, then an alternative definition has to be chosen, such as

$$R_d = \frac{1}{\gamma_R} R(f_{cR}; f_{yR}; f_{tR}; f_{p0,1R}; f_{pR}) . \tag{7-121}$$

Here, current work is being carried out. The reader should consult recent publications, such as Cervenka (2007), Allaix et al. (2007), Holicky (2007), and especially Pfeiffer and Quast (2003).

7.3.2.2 Design of partial safety factors

Several methods exist to develop partial safety factors for a certain material or a certain type of structure. But in general, all procedures rely on a statistical description of the material inherent uncertainty. Additionally, historical partial safety factors remain valid if they have proven to provide safe structures.

First, a historical procedure is introduced that permits the development of partial safety factors for concrete only on the coefficient of variation, an assumption about the type of probability distribution function, and the class of the structure (Murzewski 1974) (Tables 7-19 and 7-20). Usually the coefficient of variations for concrete depending on the production conditions lies around 10% (Spaethe 1992, Östlund 1991).

Table 7-19. Partial safety factor for a lognormal distributed strength

Class of structure	Coefficient of variation			
Class of structure	0.05	0.10	0.15	0.20
Dam	1.26	1.58	1.95	2.43
Bridge, theatre, cultural buildings	1.23	1.49	1.82	2.16
Residential buildings, office buildings	1.20	1.40	1.65	1.91
Lager, bunker, frames	1.15	1.31	1.46	1.62
Secondary buildings	1.10	1.17	1.22	1.25

Table 7-20. Partial safety factor for a normal distributed strength

	Coefficient of variation			
Class of structure	0.05	0.10	0.15	0.20
Dam	1.24–1.30	1.46–1.85		
Bridge, theatre, cultural buildings	1.21–1.26	1.41–1.67	1.60–2.47	
Residential or office buildings	1.18–1.22	1.35–1.53	1.50–2.00	1.65–2.86

	Coefficient of variation			
Lager, bunker, frames	1.15–1.16	1.27–1.37	1.38–1.61	1.49–1.92
Secondary buildings	1.10–1.11	1.17–1.19	1.21–1.25	1.23–1.28

The Eurocode (EN) is heavily based on works and suggestions by the Joint Committee of Structural Safety (JCSS). In the background document of the Eurocode provided by the JCSS (2004), an example for the development of a partial safety factor is presented. The general description for such a factor is

$$R = \Theta \cdot a \cdot x \tag{7-122}$$

with Θ as uncertainty factor for the calculation model, a as geometrical factor, and x as material strength. The resistance design value is then

$$R_d = \frac{\Theta_k \cdot a_k \cdot x_k}{\gamma_M} . \tag{7-123}$$

The three parameters might then be considered as independent random variables with a lognormal distribution. Please note that this is not true for many materials. The material partial safety factor can then be evaluated according to

$$\gamma_M = \exp(\alpha_R \cdot \beta \cdot (V_x + 0.4 \cdot V_a + 0,4 \cdot V_\Theta) - 1.64 \cdot V_x) . \tag{7-124}$$

Additionally, simplified rules exist for the development of design values, including material partial safety factors based on testing of materials. Such procedures have been intensively discussed by Reid (1999) and have been applied for masonry (Curbach and Proske 2004). The procedures include some main assumptions; for example, the probability distribution function of the load and the resistance. Examples are the Australian Standard Procedure for Statistical Proof Loading, Australian Standard Procedure for Probabilistic Load Testing, or the Standard for Probabilistic Load tests. Most of the procedures consider the variance of the material strength, the variance of the load, some condition factors, the number of tests or a correction factor, and the required safety index.

Val and Stewart (2002) describe a procedure for the evaluation of partial safety factors for mainly the resistance of existing structures. The safety factor is split into two parts, the partial safety factor for the material strength and a factor for the consideration of additional uncertainties, $f_{\mathrm{k,est}}$ is the estimated characteristic material strength. The design value for the strength is therefore

$$f_d = \frac{f_{k,est}}{\gamma_m \cdot \gamma_\eta}. \tag{7-125}$$

Tables 7-21 and 7-22 show some examples taken from the publication by Val and Stewart (2002). In Table 7-21, no *a priori* information was available, whereas in Table 7-22, *a priori* information could be used. It becomes clear that additional information is for existing structures of major importance for keeping safe and efficient structures. As already seen in the JCSS (2004) example, the consideration of modelling uncertainty, in which ever way, is a major part of many expansions of the traditional ways for the developing partial safety factors. Here it becomes visible that the clear rule for the application of statistics might sometimes be a drawback for a realistic estimation of the safety of a structure.

Table 7-21. Example of material partial safety factors without *a priori* and with six tested samples (Val and Stewart 2002)

Coefficient of variation of the samples	γ_m	Coefficient of variation for the calibration factor 0.05	0.10	0.15
0.04	1.1	1.07	1.19	1.27
0.08	1.4	1.05	1.17	1.24
0.12	1.6	1.04	1.15	1.21
0.16	1.9	1.03	1.10	1.19
0.20	2.2	1.02	1.07	1.15

Table 7-22. Example of material partial safety factors with *a priori* and with six tested samples (Val and Stewart 2002)

A priori coefficient of variation	γ_m
0.05	1.36
0.10	1.25
0.15	1.21

Melchers and Faber (2001) introduce a damage function, which is included in the following material safety factor:

$$\gamma_M = \gamma_{M0} \cdot \varphi_P \cdot \varphi_D \cdot \varphi_M. \tag{7-126}$$

The factors consider deterioration processes, protection against deterioration, and intensity of inspection.

Schneider (1999) has also investigated the development of partial safety factors. Partial safety factors for existing reinforced concrete structures have recently been discussed by Fischer and Schnell (2008) and Braml and Keuser (2008).

Besides the simplified techniques, the partial safety factor can also be computed using the results from full probabilistic computations (FORM). Then, the so-called weighting factors α_R of the random variables are used. The partial safety factor for material strength γ_m is computed as

$$\gamma_m = \frac{R_c}{R_d}. \tag{7-127}$$

The safety factor can be computed when design resistance R_d and characteristic resistance R_c are known. Since

$$R_d = \mu_E - \alpha_R \cdot \beta \cdot \sigma_R \tag{7-128}$$

and for a normal distribution with known mean value and standard deviation, the characteristic value is given as

$$R_c = \mu_E - 1.645 \cdot \sigma_R, \tag{7-129}$$

hence the partial safety factor for the material strength can be computed. As an example, Fig. 7-13 shows the distribution of the weighting factor quadrates for historical arch bridges, either for road or for railway traffic based on FORM computations. This is a common type of diagram since the quadrates of the weighting factors have to sum up to one. The figure clearly shows why in railway codes the partial safety factor for the traffic load is often lower; for example, 1.3 compared to road traffic bridges with 1.5. Here it can be seen that the weighting factor is significantly lower for railway traffic than for road traffic.

Table 7-23 shows the computation of partial safety factors for live and dead load subjected to an adaptable goal safety index for arch bridges. Table 7-24 lists system partial safety factors for stone arch bridges subject to different masonry types. The investigation is based on own probabilistic computations.

In general, the issue of partial safety factors for historical stone arch bridges is still under discussion since they show a highly nonlinear behaviour and such structures can only be understood as a system and not on the cross-section layer.

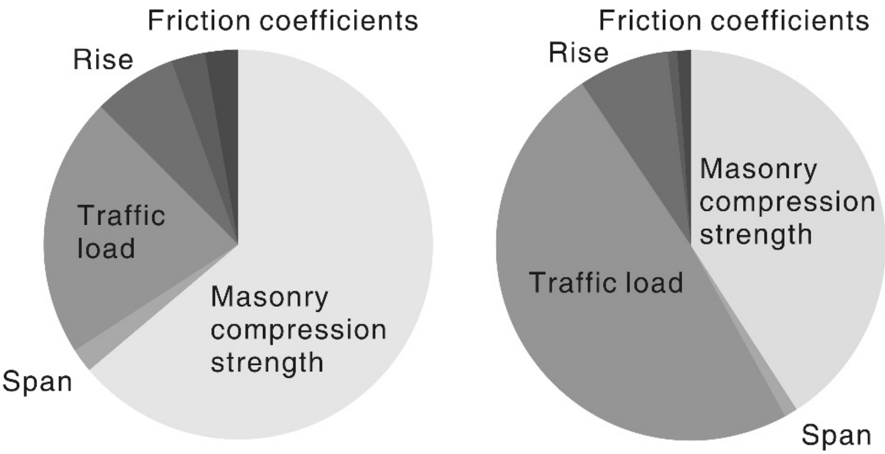
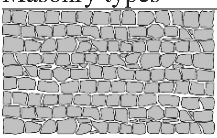
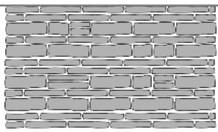
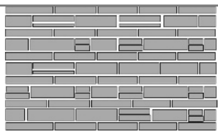
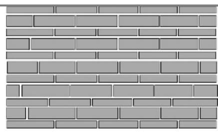


Fig. 7-13. Distribution of the square of the weighting factors for an arch bridge using a linear-elastic model for railway traffic (*left*) and road traffic (*right*)

Table 7-23. Partial safety factor for dead and live load considering the adaptation of the safety index according to Schueremans and Van Gemert (2001)

$\Delta_S + \Delta_R + \Delta_P + \Delta_I \leq 1.5$	Partial safety factor for		Factor of combination
	Dead load	Live load	
–0.25	1.42	1.56	0.69
0.00	1.35	1.50	0.70
0.25	1.28	1.45	0.71
0.50	1.22	1.39	0.72
0.75	1.16	1.34	0.73
1.00	1.10	1.29	0.74
1.25	1.05	1.25	0.75
1.50	1.00	1.20	0.76

Table 7-24. Own suggestions for partial safety factors for historical masonry arch bridges subject to different masonry types in the arch

Masonry types			
			
System partial safety factor			
$\gamma_M = 2.1$	$\gamma_M = 2.0$	$\gamma_M = 1.9$	$\gamma_M = 1.8$

7.3.3 Characteristic Values

Characteristic values are probability fixed defined values of a probability distribution. They relate the probability distribution property—for example, strength—to a certain probability. A characteristic strength value f_k of structural materials of 5% of the overall population is very common. That means that only 5% of the overall population will experience a lesser strength, and 95% of the population will show a higher strength than this 5% fractile material strength. The terms percentile and quantile can often also be found in literature instead of fractile. The chosen value of 5% is arbitrary, but the general idea about characteristic values are the proofs in the state of serviceability carried out without partial safety factor (one). Since characteristic values and partial safety factors interact, the characteristic value simply has to be chosen to provide the partial safety factor of one for the limit state of serviceability. Therefore, the assumption of the 5% fractile value can be found as a general requirement, for example, in the Eurocode 1 (1994) or in the German DIN 1055-100, Sect. 6.4 (1999). As an example, some material-related codes are listed here to show the wide application of the 5% fractile value assumption:

- Reinforcement steel according to DIN 488 (90% confidence interval)
- Concrete compression strength according to DIN 1045, DIN 1048
- Masonry after testing according to DIN 1053 (75% confidence interval) (Schubert 1995)
- Outer wall panelling according to DIN 18516 (75% confidence interval)
- Masonry according to DIN 18152 (1987) with 90% confidence interval
- Reinforcement steel according to ENV 100080 (90% confidence interval)
- Wooden structures according to DIN V ENV 1995 (84,1%-confidence interval, coefficient of variation greater or equal 0.1, more than 30 samples)
- Masonry-aerated concrete (95%-confidence interval)
- Artificial brick stones (Schubert 1995)
- Natural stones (90% confidence interval) (Schubert 1995)

To compute a 5% fractile value of a certain property, statistical information is required. Using such statistical information, a probability distribution function can be estimated. There exists a wide variety of such probability functions as shown in Table 7-25. Many distribution functions can be related to each other (Fig. 7-14). Other distribution functions describe certain distribution families or certain conditions; for example, Piersons differental equation and the Fleishmann system. An overview of distribution families can be found in Plate (1993), Bobee and Ashkar (1991), and Fischer (1999). Besides that for extreme value distributions, the work by Castillo et al. (2008) is mentioned. For practical reasons, however, only a limited number of distributions is considered for construction material: the normal distribution, the lognormal distribution, and the Weibull distribution (Fischer 1999, Eurocode 1 1994, and GruSiBau 1981). The normal distribution has found wide application and can be easily explained by the central limit state theorem. This theorem states that a sum of certain random variables will yield to a normal distributed random variable if certain conditions are fulfilled (Van der Werden 1957). Such conditions are, for example, that no single random variable dominates the results. And indeed, many material properties can be seen as the sum of certain other properties—for example, the strength of a natural stone can be seen as sum of the strength of the single elements of the stone. Therefore, many publications assume a normal distribution for the concrete compression strength (Rüsch et al. 1969). A disadvantage of the normal distribution is possible negative values. Therefore, instead of the normal distribution, the lognormal distribution is often used if the average value of a material property is low and experiences a high standard deviation, such as the tensile strength of concrete or masonry. The lognormal distribution does not feature negative values and therefore no negative tensile strengths are then possible. The lognormal distribution can also be related to the central limit state theorem, if the data are logarithmic. However, this implies the multiplication of the single input random variables—in other terms, the logarithmic distribution fulfils the central limit state theorem for the case of multiplication. A further often-used distribution for construction material properties is the Weibull distribution (Weibull 1951). This distribution belongs to the group of extreme value distributions and describes a chain interaction of single elements. If the weakest part of the chain fails, then the entire chain fails, which indeed fulfil's the requirements of an extreme value distribution. The properties of brittle materials, such as glass, can be described with this distribution (Button et al. 1993, Güsgen et al. 1998). Of course, many materials do not comply with the chain rule for a serial system but show rather a mixed parallel-serial system. Further considerations are then needed. Some theoretical works dealing with this issue have been taken out by Rackwitz and Hohenbichler (1981), Gollwitzer and Rackwitz (1990), and Kadarpa et al. (1996) for brittle materials; Chudoba et al. (2006) and Chudoba and Vorechovsky (2006) for glass yarns.

Table 7-25. Certain probability functions

	Name of distribution		Name of distribution
1	χ-Distribution	29	Laplace distribution
2	General Pareto distribution	30	Logarithmic Pearson typ-3 distribution
3	Arcsin distribution	31	Logarithmic–logistic distribution
4	Beta distribution	32	Logistic distribution
5	Binomial distribution	33	Lognormal distribution
6	Birnbaum–Saunders distribution	34	Lorenz distribution
7	Breit–Wigner distribution	35	Maxwell distribution
8	Cauchy distribution	36	Neville distribution
9	Erlang distribution	37	Pareto distribution
10	Exponential distribution	38	Pearson, typ-3, gamma distribution
11	Extreme value distribution typ I max	39	Pearson, typ-3 distribution
12	Extreme value distribution typ I min	40	Poisson distribution
13	Extreme value distribution typ II max	41	Polya distribution
14	Extreme value distribution typ II min	42	Potential distribution
15	Extreme value distribution typ III max	43	Power normal distribution
16	Extreme value distribution typ III min	44	Rayleigh distribution
17	Fisher distribution	45	Uniform distribution
18	Fréchet distribution	46	Reverse Weibull distribution
19	F distribution	47	Rossi distribution
20	Gamma distribution (G-distribution)	48	Simpson or triangle distribution
21	Gauss order normal distribution	49	Sinus distribution
22	General extreme value distribution	50	Snedecor distribution
23	General Pareto distribution	51	Student-t-distribution
24	Geometric distribution	52	Tukey Lambda distribution
25	Gumbel distribution	53	Wakeby distribution
26	Hypergeometric distribution	54	Weibull distribution
27	Krickij–Menkel distribution	55	Wishart distribution
28	Landau distribution	56	Z-Distribution

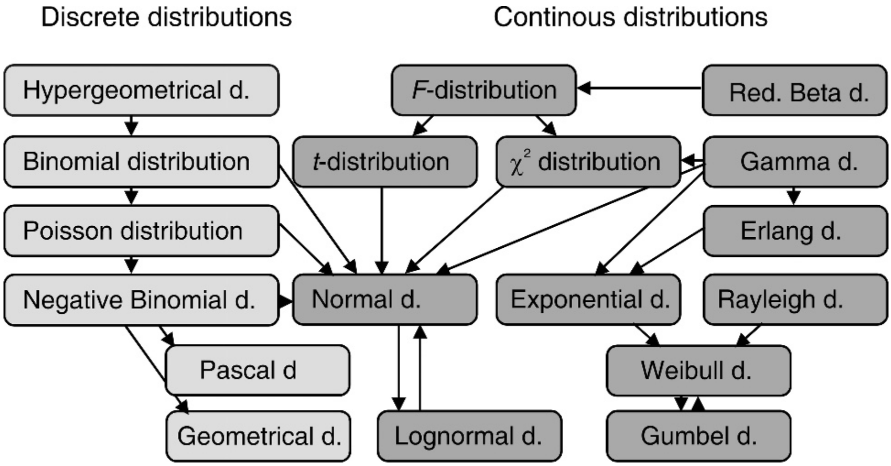


Fig. 7-14. Distribution families according to Fischer (1999)

However, theoretical considerations often do not prove the type of probability distribution. Therefore, usually the statistical data are investigated, according to DIN 53804 (1981). The most robust and fast converting parameters of the random data are general tendency estimators such as the arithmetical mean, harmonically mean, geometrical mean, generalized mean, quadratic mean and median (50% fractile), and mode (most probable value). However, more interesting from a statistical point of view, are deviation measures or uncertainty measures such as variance, standard deviation (same unit as mean value), coefficient of variation, span, modified interquartile range, and mean deviation. Besides such measures, the skewness and the kurtosis are also of interest (Fig. 7-15).

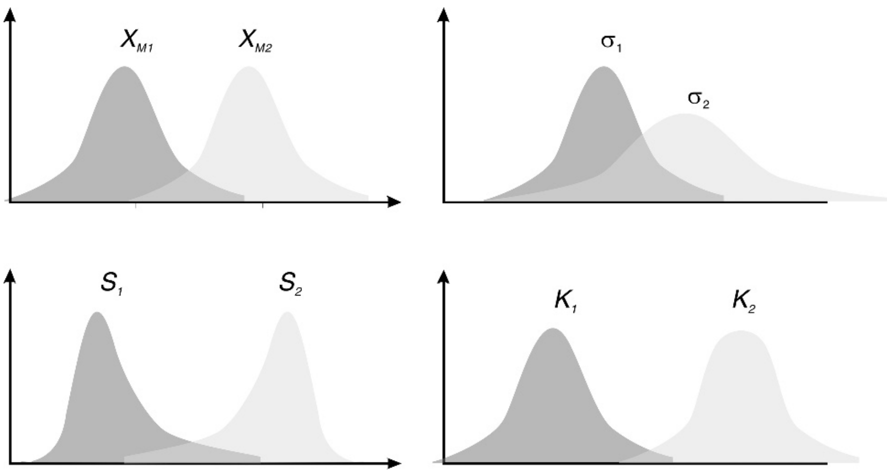


Fig. 7-15. Meaning of the statistical parameter arithmetic mean (X_m), standard deviation (σ), skewness (S), and kurtosis (K)

For historical masonry, one should keep in mind that such statistical parameters may be insufficient due to corrupted data (Fig. 7-16). Statistical data from historical masonry with natural stone may feature outliers, censored data, and multimodal data. Outliers are samples that do not belong to the population. However, since we do not know the population, it is difficult to identify outliers. Here, Dixons test, Barnett–Lewis test, Chauvenets criteria, Grupps test, the David-Hartley-Pearson test and other criteria may be used for outlier identification (McBean and Rovers 1998, Fischer 1999, and Bartsch 1991). Censored data describe a condition in which access to the original population data is filtered by a process. For example, investigating the strength of historical mortar may yield to censored data since the drilling process using high-pressure cooling water may destroy mortar inside the masonry. Furthermore, sawing the test specimen may further destroy material. Therefore, the compression test results of the mortar may indicate high average compression strength, however the tests do not consider the failure of all weak material in the preparation process. Therefore, the data are censored and have to be corrected. Then, for example, Cohens and Aitchison's method can be applied for data correction (McBean and Rovers 1998). Finally, it may be the case that the test results do not come from one population. For example, different stone types may be used for masonry or the natural stone material may come from different stone quarries. Then the compression strength can show multimodal behaviour. Using optimization methods, it is possible to disintegrate different distributions from such multimodal data (Proske 2003).

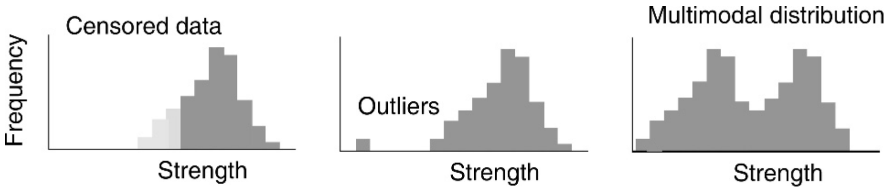


Fig. 7-16. Examples of corrupted data

After the evaluation of the single parameters, the probability distribution function usually has to be chosen. Several statistical techniques exist to investigate statistical data and recommend a distribution type:

- Relating the coefficient of variance and the type of distribution
- Relating skewness and kurtosis and the type of distribution (Fig. 7-17)
- The minimum sum square error based on histograms
- The χ^2 test and $n\omega^2$ test
- The Kolmogoroff-Smirnoff test
- The Shapiro-Wilk or Shapiro-Francia test

- Probability plots
- Quantile-correlation values

Such statistical tests are often called goodness-of-fit tests. However, as described, statistical testing is only one part of the investigation. The understanding of the material extraction, the testing techniques, and the material itself is compelling to interpret the results of material testing and statistical investigations.

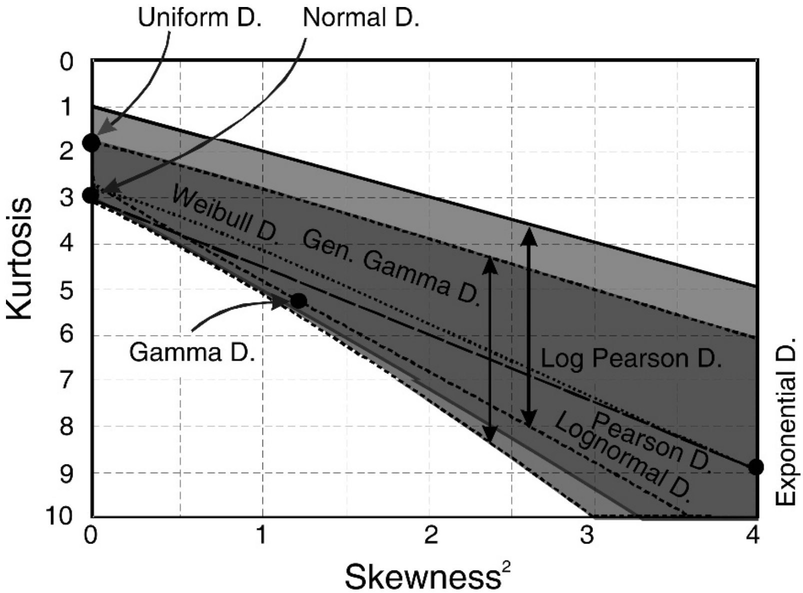


Fig. 7-17. Relation between skewness and kurtosis and the type of distribution (Plate 1993) D. = distribution

Besides the behaviour of single random variables, correlation between the random variables is often of great interest in identifying deterministic formulas. However, the identification of different correlation coefficients is rather laborious due to the required high sample sizes. Figure 7-18 shows that measuring a correlation coefficient of 0.5 for ten samples shows a value for the population between –0.5 and 0.8. Furthermore, Pearson's coefficient of correlation may not be adequate, other correlation coefficients such as from Spearman, Kendall, and Hoeffding may be helpful. Additionally, nonlinear regression using the Levenberg-Marquardt method can be considered.

Besides such limitations, the next sections will discuss the computation of characteristic 5% fractile values for certain probability distribution functions.

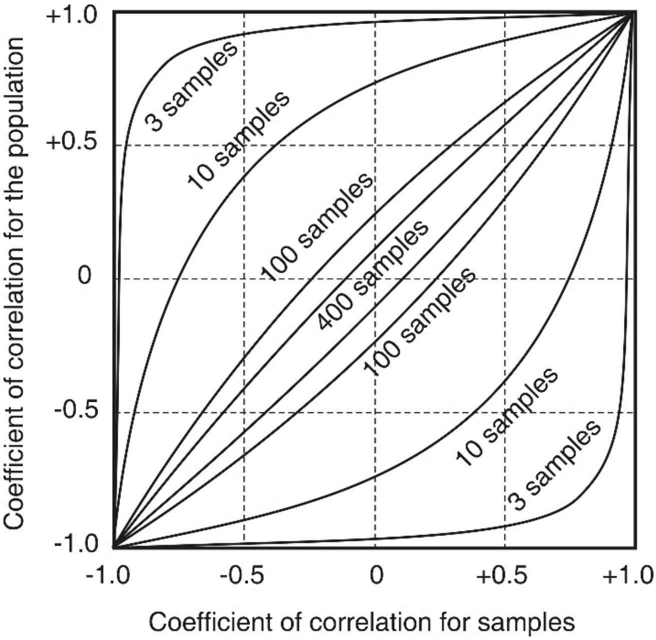


Fig. 7-18. Ninty-five percent confidence interval for coefficient of correlations (Steel and Torrie 1991)

7.3.3.1 Normal distribution

The 5% fractile value f_k of a normal distributed material strength can be computed as (Storm 1988)

$$f_k = f_m - k \cdot \sigma \,, \tag{7-130}$$

where f_m is the mean value and σ is the standard deviation. If the mean value and the standard deviation are known, then the *k*-factor corresponding to the 5% fractile value amounts to 1.645. However, usually only some suggestions for the mean value and the standard deviation are known due to a limited number of samples. Although the mean value converges very fast with a low number of samples, the problem remains for the evaluation of the standard deviation. The uncertainty in the empirical statistical parameters is usually considered in the choice of the *k*-factor.

Assuming, for example, a normal distributed property with 15 samples and a confidence interval of 95%, the *k*-factor becomes 1.76 based on a student-t distribution (Table 7-26). For very high sample numbers, the *k*-value converges to 1.645 again.

Table 7-26. *k*-factors based on a student's *t*-distribution (degrees of freedom = sample number – 1)

Degree of freedom	Fractile value 5%	2.5%	Degrees of freedom	Fractile value 5%	2.5%
1	6.314	12.706	20	1.725	2.086
2	2.920	4.303	21	1.721	2.080
3	2.353	3.182	22	1.717	2.074
4	2.132	2.776	23	1.714	2.069
5	2.015	2.571	24	1.711	2.064
6	1.943	2.447	25	1.708	2.060
7	1.895	2.365	26	1.706	2.056
8	1.860	2.306	27	1.703	2.052
9	1.833	2.262	28	1.701	2.048
10	1.812	2.228	29	1.699	2.045
11	1.796	2.201	30	1.697	2.042
12	1.782	2.179	40	1.684	2.021
13	1.771	2.160	60	1.671	2.000
14	1.761	2.145	80	1.664	1.990
15	1.753	2.131	100	1.660	1.984
16	1.746	2.120	200	1.653	1.972
17	1.740	2.110	500	1.648	1.965
18	1.734	2.101	1000	1.646	1.962

The Eurocode 1 suggests slightly different *k*-factors. For example, according to the Eurocode 1, 15 samples would yield a *k*-factor of 1.84.

A simple example should illustrate the application of a normal distribution. About 500 compression test results of Posta sandstone are available for an investigation. The mean value of the compression strength is 58.05 MPa and the standard deviation is given with 10.32 MPa. Assuming a normal distribution for 500 samples, the *k*-value becomes 1.648. The characteristic compression strength then is computed as

$$f_{st,k} = 58.05\ \text{MPa} - 1.648 \cdot 10.32\ \text{MPa} = 41.04\ \text{MPa}\ . \tag{7-131}$$

This value is compared with other references discussing the compression strength of Posta sandstone. Since the sample size is unknown, the *k*-value is kept constant with 1.645.

Grunert (1982) $f_{st,k} = f_m - 1.645 \cdot \sigma = 45.6 - 1.645 \cdot 11.6 = 26.52\ \text{MPa}$

Grunert et al. (1998) $f_{st,k} = f_m - 1.645 \cdot \sigma = 31.6 - 1.645 \cdot 6.8 = 20.41\ \text{MPa}$

Peschel (1984) $f_{st,k} = f_m - 1.645 \cdot \sigma = 41.6 - 1.645 \cdot 11.6 = 22.52\ \text{MPa}$

7.3.3.2 Lognormal distribution

As mentioned above, the lognormal distribution can be reasoned based on a product formulation of the central limit theorem. The computation of the 5% fractile value is analogous to the normal distribution, except that the data have to be transformed by the logarithm:

$$L' = \overline{y} - k \cdot s^* = \ln(L) \tag{7-132}$$

$$\overline{y} = \sum_{i=1}^{n} \frac{y_i}{n} \text{ with } y_i = \ln(x_i) \tag{7-133}$$

$$s^* = \sqrt{\frac{\sum_{i=1}^{n} \left(y_i - \overline{y}\right)^2}{n-1}} \,. \tag{7-134}$$

The following example should show the application. Four compression strength tests from natural stone masonry have been carried out (Table 7-27).

Table 7-27. Test data

Sample	Compression strength in MPa	Logarithm
1	6.70	1.90
2	6.20	1.82
3	5.70	1.74
4	6.60	1.89
Mean value	6.30	1.84
Standard deviation	0.39	0.06

The characteristic value assuming a lognormal distribution can be computed as follows (for 2.353 see Table 7-26):

$$f_{mw,k} = \exp(1.84 - 2.353 \cdot 0.06) = 5.47 \text{ MPa} \,. \tag{7-135}$$

If a normal distribution is assumed, then the characteristic value becomes

$$f_{mw,k} = 6.30 \text{ MPa} - 2.353 \cdot 0.39 \text{ MPa} = 5.37 \text{ MPa} \,. \tag{7-136}$$

7.3.3.3 Weibull distribution

For the Weibull distribution, the 5% fractile value can be estimated with

$$f_k = \left(-\frac{1}{\lambda} \ln(1-q) \right)^{\frac{1}{k}} . \quad (7\text{-}137)$$

The value q describes the probability that is chosen in our case as 0.05 for the 5% fractile value. The factors λ and k are parameters of the Weibull distribution, which can be computed based on statistical data. The mean value and the standard deviation can be computed as

$$f_m = f_0 + \lambda^{-1/k} \cdot \Gamma\left(1 + \frac{1}{k}\right), \quad (7\text{-}138)$$

$$\sigma = \lambda^{-1/k} \sqrt{\Gamma\left(1 + \frac{2}{k}\right) - \Gamma\left(1 + \frac{1}{k}\right)^2} . \quad (7\text{-}139)$$

Since the mean value and the standard deviation of the test data can usually be easily computed, these formulas are useful in computing the factors λ and k.

Again, an example shows the application. Consider the test results of the compression strength of natural stone masonry shown in Table 7-27. Using the mean value of 6.30 MPa and the standard deviation of 0.39 MPa, the k value can be computed with 20.01 and the λ becomes 5.84×10^{-17}. The 5% fractile value is then given as

$$f_k = \left(-\frac{1}{5.84 \cdot 10^{-17}} \ln(0.95) \right)^{\frac{1}{20.01}} = 5.58 \text{ MPa} . \quad (7\text{-}140)$$

7.3.3.4 Leicester method

The Leicester method estimates the 5% fractile value without the selection of a probability function (Hunt and Bryant 1996). The 5% fractile value is computed as

$$f_k = A \cdot \left(1 - \frac{2.7 \cdot v}{n}\right) \quad (7\text{-}141)$$

with

n as number of samples preferred higher than 30

v as coefficient of variation preferred smaller than 0.5

A as empirical 5% fractile of the data. This value is often computed by linear interpolation.

The simple application is shown in the following example. Using 500 stone compression strength tests, the empirical 5% fractile value of the sample data is estimated with 42.3 MPa. Such estimations can, for example, be done with many spreadsheet programs such as Excel. In Excel, the "rang and quantile" function can be used. The coefficient of variation is 0.177. Then, the corrected 5% fractile value is given by

$$f_k = 42.3\ \text{MPa} \cdot \left(1 - \frac{2.7 \cdot 0.177}{500}\right) = 42.26\ \text{MPa}\ . \tag{7-142}$$

7.3.3.5 Öferbeck method

A further technique is the Öfverbeck Power Limit (Hunt and Bryant 1996). Using the constant ε, subject to the number of samples and the number of used samples *q,* the 5% fractile value is given as

$$f_k = x_q^{1-\varepsilon} \prod_{i=1}^{q-1} x_i^{\varepsilon/(q-1)}\ . \tag{7-143}$$

The relevant constants are given in Table 7-28.

Table 7-28. Relevant constants *q* and ε subject to the overall sample size

Sample size *n*	Number of used samples *q*	Öfverbeck constant ε
5	2	5.93
6	2	5.35
7	2	4.85
8	2	4.42
9	2	4.03
10	3	3.31
11	3	3.12
12	3	2.96
13	3	2.80
14	3	2.66
15	3	2.53
20	4	2.22
30	5	1.80
40	6	1.58
50	7	1.44

Again, the data of masonry compression strength are used as illustration. Only four samples are available. Using Table 7-28, *q* is 2 and 6.00 is chosen for ε by extrapolation. The samples are given by 5.7, 6.2, 6.6, and 6.7 MPa. The 5% fractile value is then

$$f_k = x_q^{1-\varepsilon} \prod_{i=1}^{q-1} x_i^{\varepsilon/(q-1)} = x_2^{1-6.00} \prod_{i=1}^{2-1} x_i \tag{7-144}$$

$$= 6.2^{1-6} \cdot 5.7^{6/(2-1)} = \frac{5.7^6}{6.2^5} = 3.74 \text{ MPa} .$$

7.3.3.6 Jaeger and Bakht method

Jaeger and Bakht (1990) present a technique to estimate the 5% fractile based on a combination of different probability functions. They call this artificial function a log arc sinh normal polynomial distribution. Assuming a quadratic polynomial in the distribution, and the consideration of only the lowest, average and highest test value, the fractile value can be computed with the following steps:

1. Sort the data.
2. Compute the mean value. If the sample number is uneven, then use the median. If the sample number (n=2 × k) is even, use the following formula:

$$f_m = \frac{(f_k + f_{k+1})}{2} . \tag{7-145}$$

3. Transform the data according to

$$y_i = \frac{f_i^2 - f_m^2}{2 \cdot f_i \cdot f_m} . \tag{7-146}$$

4. Chose the characteristic value—in our case, the 5% fractile value. Then chose the representing k-value of a normal distribution, here $k = z^* = -1,645$.
5. Chose a z_1 according to Table 7-29.

Table 7-29. Representative z_1 value subject to the sample number

Sample number n	z_1	
10	1.34	
11	1.38	As approximation, the following equation
12	1.43	can be recommended:
13	1.47	$z_1 = -0.8004 - 0.0649 \cdot n + 0.0011 \cdot n^2$
14	1.50	
15	1.53	
16	1.56	

Sample number n	z_1
17	1.59
18	1.62
19	1.65
20	1.67

6. Compute the 5% fractile value for the transformed data:

$$y^* = \left(\frac{y_1 - y_n}{2}\right)\left(\frac{z^*}{z_1}\right) + \left(\frac{y_1 + y_n}{2}\right)\left(\frac{z^*}{z_1}\right)^2 . \tag{7-147}$$

7. Compute the 5% fractile value for the original data:

$$f_k = f_m (y^* + \sqrt{1 + (y^*)^2} . \tag{7-148}$$

The introduced list is only one possible technique mentioned by Jaeger and Bakht (1990); however, they consider this one as the numerically most robust one. The other techniques consider other data points, such as the smallest, the second smallest, and the mean value, or other orders of the polynomial, and yield slightly different 5% fractile values.

Again, an example illustrates the approach. Fifteen bending test results of granite stones are used. The test results are then related to one specimen height and listed in Table 7-30.

Table 7-30. Test results and transformed data

No.	Bending tensile strength in MPa	Percent	y_i
1	14.710	100.00	0.35106
2	12.802	92.80	0.20674
3	12.700	85.70	0.19858
4	12.506	78.50	0.18291
5	11.150	71.40	0.06719
6	10.829	64.20	0.03793
7	10.822	57.10	0.03729
8	10.426	50.00	0.00000
9	10.208	42.80	–0.02113
10	10.002	35.70	–0.04153
11	9.919	28.50	–0.04987
12	9.664	21.40	–0.07597
13	9.412	14.20	–0.10250
14	6.281	7.10	–0.52875
15	4.233	0.00	–1.02851

According to Step 2, the mean value is given as f_m=10.426 MPa. The transformed data (Step 3) are already included in Table 7-30. Furthermore, the 5% fractile value should be computed, therefore $z^* = -1.645$ and z_1= 1.53. Then the transformed 5% fractile value is given as

$$y^* = \left(\frac{-1.029-0.351}{2}\right)\left(\frac{-1.645}{-1.53}\right)+\left(\frac{-1.029+0.351}{2}\right)\left(\frac{-1.645}{-1.53}\right)^2 \quad (7\text{-}149)$$

$$y^* = -1.133$$

and for the original data

$$f_k = 10.426 \text{ MPa}\cdot\left(-1.133+\sqrt{1+(-1.133)^2}\right) = 3.94 \text{ MPa}\,. \quad (7\text{-}150)$$

7.3.3.7 Binomial distribution

Mehdianpour (2006) has used diagrams of binomal distribution to estimate characteristic compression strength values. Details can be found in the original reference.

All mentioned techniques consider the uncertainty of material and loading data in a stochastic way. However, such a consideration does not necessarily end in the preparation of fractile values and partial safety factors. Increasingly, full probabilistic computations of structures and also of historical stone arch bridges can be found in the literature. Therefore, Chapter 8 discusses the results of those numerical investigations.

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