14.2 PERIODIC AND OSCILLATORY MOTIONS

Fig 14.1 shows some periodic motions. Suppose an insect climbs up a ramp and falls down it comes back to the initial point and repeats the process identically. If you draw a graph of its height above the ground versus time, it would look something like Fig. 14.1 (a). If a child climbs up a step, comes down, and repeats the process, its height above the ground would look like that in Fig 14.1 (b). When you play the game of bouncing a ball off the ground, between your palm and the ground, its height versus time graph would look like the one in Fig 14.1 (c). Note that both the curved parts in Fig 14.1 (c) are sections of a parabola given by the Newton's equation of motion (see section 3.6),

$$h = ut + \frac{1}{2}gt^2 \text{ for downward motion, and}$$

$$h = ut - \frac{1}{2}gt^2 \text{ for upward motion,}$$

with different values of u in each case. These are examples of periodic motion. Thus, a motion that repeats itself at regular intervals of time is called **periodic motion.**

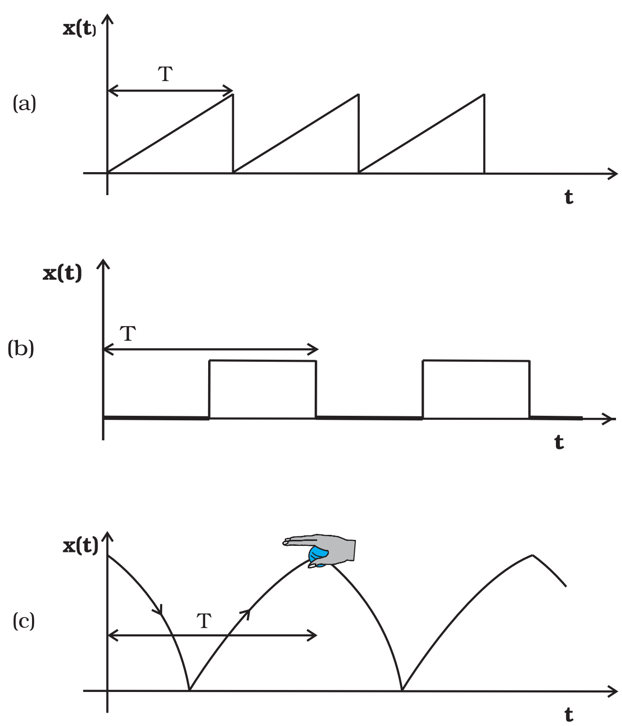

Fig 14.1 *Examples of periodic motion. The period T is shown in each case.*

Very often the body undergoing periodic motion has an equilibrium position somewhere inside its path. When the body is at this position no net external force acts on it. Therefore, if it is left there at rest, it remains there forever. If the body is given a small displacement from the position, a force comes into play which tries to bring the body back to the equilibrium point, giving rise to **oscillations** or **vibrations**. For example, a ball placed in a bowl will be in equilibrium at the bottom. If displaced a little from the point, it will perform oscillations in the bowl. Every oscillatory motion is periodic, but every periodic motion need not be oscillatory. Circular motion is a periodic motion, but it is not oscillatory.

There is no significant difference between oscillations and vibrations. It seems that when the frequency is small, we call it oscillation (like the oscillation of a branch of a tree), while when the frequency is high, we call it vibration (like the vibration of a string of a musical instrument).

Simple harmonic motion is the simplest form of oscillatory motion. This motion arises when the force on the oscillating body is directly proportional to its displacement from the mean position, which is also the equilibrium position. Further, at any point in its oscillation, this force is directed towards the mean position.

In practice, oscillating bodies eventually come to rest at their equilibrium positions, because of the damping due to friction and other dissipative causes. However, they can be forced to remain oscillating by means of some external periodic agency. We discuss the phenomena of damped and forced oscillations later in the chapter.

Any material medium can be pictured as a collection of a large number of coupled oscillators. The collective oscillations of the constituents of a medium manifest themselves as waves. Examples of waves include water waves, seismic waves, electromagnetic waves. We shall study the wave phenomenon in the next chapter.

14.2.1 Period and frequency

We have seen that any motion that repeats itself at regular intervals of time is called **periodic motion**. **The smallest interval of time after which the motion is repeated is called its period**. Let us denote the period by the symbol T. Its SI unit is second. For periodic motions, which are either too fast or too slow on the scale of seconds, other convenient units of time are used. The period of vibrations of a quartz crystal is expressed in units of microseconds (10^{-6} s) abbreviated as μs. On the other hand, the orbital period of the planet Mercury is 88 earth days. The Halley's comet appears after every 76 years.

The reciprocal of T gives the number of repetitions that occur per unit time. This quantity is called the **frequency of the periodic motion**. It is represented by the symbol ν . The relation between υ and T is

$$\upsilon = 1/T \tag{14.1}$$

The unit of ν is thus s^{-1}. After the discoverer of radio waves, Heinrich Rudolph Hertz (1857-1894), a special name has been given to the unit of frequency. It is called hertz (abbreviated as Hz). Thus,

$$1 \text{ hertz} = 1 \text{ Hz} = 1 \text{ oscillation per second} = 1 s^{-1} \tag{14.2}$$

Note, that the frequency, ν, is not necessarily an integer.

Example 14.1 On an average a human heart is found to beat 75 times in a minute. Calculate its frequency and period.

Answer The beat frequency of heart = 75/(1 min)
= 75/(60 s)
= 1.25 s^{-1}
= 1.25 Hz
The time period T = 1/(1.25 s^{-1})
= 0.8 s

14.2.2 Displacement

In section 4.2, we defined displacement of a particle as the change in its position vector. In this chapter, we use the term displacement in a more general sense. It refers to change with time of any physical property under consideration. For example, in case of rectilinear motion of a steel ball on a surface, the distance from the starting point as a function of time is its position displacement. The choice of origin is a matter of convenience. Consider a block attached to a spring, the other end of which is fixed to a rigid wall [see Fig.14.2(a)]. Generally it is convenient to measure displacement of the body from its equilibrium position. For an oscillating simple pendulum, the angle from the vertical as a function of time may be regarded as a displacement variable [see Fig.14.2(b)]. The term displacement is not always to be referred in the context of position only. There can be many other kinds of displacement variables. The voltage across a capacitor, changing with time in an a.c. circuit, is also a displacement variable. In the same way, pressure variations in time in the propagation of sound wave, the changing electric and magnetic fields in a light wave are examples of displacement in different contexts. The displacement variable may take both positive and negative values. In experiments on oscillations, the displacement is measured for different times.

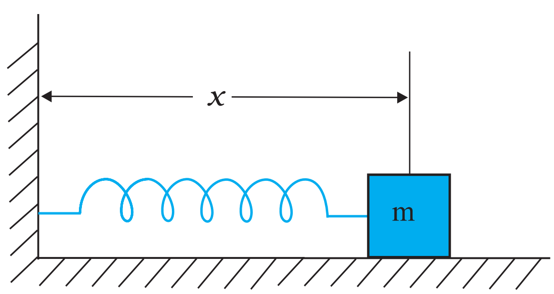

Fig. 14.2(a) *A block attached to a spring, the other end of which is fixed to a rigid wall. The block moves on a frictionless surface. The motion of the block can be described in terms of its distance or displacement x from the wall.*

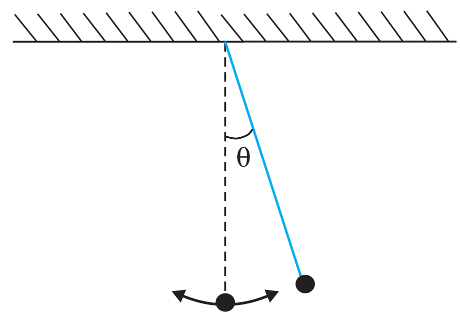

Fig.14.2(b) *An oscillating simple pendulum; its motion can be described in terms of angular displacement θ from the vertical.*

The displacement can be represented by a mathematical function of time. In case of periodic motion, this function is periodic in time. One of the simplest periodic functions is given by

$$f(t) = A \cos \omega t \tag{14.3a}$$

If the argument of this function, ωt, is increased by an integral multiple of 2π radians,